COURSE DESCRIPTION: CROSSED PRODUCTS OF C*-ALGEBRAS AND BANACH ALGEBRAS, UNIVERSITY OF TORONTO, SPRING SEMESTER 2014

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A crossed product is the functional analysts' version of a skew group ring. Thus, if $\alpha \colon G \to \mathrm{Aut}(A)$ is an action of a locally compact group G on a Banach algebra A, then a crossed product Banach algebra encodes the action of G on A, and its representation theory is related to pairs (u, π) consisting of a representation u of G and a representation π of A on the same Banach space such that $u_g \pi(a) u_g^{-1} = \pi(\alpha_g(a))$ for all $g \in G$ and $a \in A$.

There is an extensive theory of crossed product C*-algebras, with many active directions of research. Crossed product constructions are a very important way of producing new C*-algebras from old ones. By contrast, crossed products of other sorts of Banach algebras have received very little attention. Very recent work suggests that there are a number of interesting examples. There may be an interesting general theory of crossed products of algebras of operators on L^p spaces and possibly of other classes of Banach algebra crossed products.

This course will primarily be about the structure of C*-algebra and L^p operator algebra crossed products by discrete groups (although I hope to inspire interest in other classes of Banach algebra crossed products), and to some extent about the classification of simple nuclear C*-algebra arising as crossed products by $\mathbb{Z}$, $\mathbb{Z}^d$, and finite groups. I will give the basic construction and a number of examples, doing both the C* and L^p theories in parallel as far as the L^p theory currently goes. I will continue with further results in the C* theory. I will use the C* results as a source of ideas for possible new research directions for L^p operator crossed products and to some extent for other kinds of Banach algebra crossed products.

The C*-algebra part of the course will follow parts of my (still very incomplete) lecture notes [4]. (I hope to make further progress on them before the course starts, and I expect to continue to update them as the course proceeds.) I know of just one book on crossed products,

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that of Williams [6], although one might also count Pedersen's book [2] (see the last two chapters). These sources go in a somewhat different direction than the course. There is nothing on Banach algebras in my lecture notes, so I have to refer to the original preprints. The main one is [3]. A basic general theory of Banach algebra crossed products is developed in [1]; this paper corresponds to some of the early part of [6]. I also intend to present a result from [5].

Students interested in C* crossed products should have a basic background in C*-algebras, such as could be obtained from a one quarter or one semester course on C*-algebras. I will occasionally use K-theory, but not in an essential way, and knowledge of K-theory is not required. Students interested in Banach algebra crossed products should have a basic background in Banach algebras, and for the C*-algebra parts be willing to take on faith some parts of the theory of C*-algebras, especially results about positive elements and continuous functional calculus.

References

[1] S. Dirksen, M. de Jeu, and M. Wortel, *Crossed products of Banach algebras. I*, Dissertationes Math., to appear. (arXiv:1104.5151v2 [math.OA].)

[2] G. K. Pedersen, *C*-Algebras and their Automorphism Groups*, Academic Press, London, New York, San Francisco, 1979.

[3] N. C. Phillips, *Crossed products of L^p operator algebras and the K-theory of Cuntz algebras on L^p spaces*, preprint (arXiv: 1309.6406 [math.FA]).

[4] N. C. Phillips, *Crossed Product C*-Algebras and Minimal Dynamics* (lecture notes), in preparation. Current version available at: `http://pages.uoregon.edu/ncp/Courses/CRMCrPrdMinDyn/CRMCrPrdMinDyn.html`.

[5] S. Pooya and S. Hejazian, *Simplicity of reduced L^p operator crossed products with Powers groups*, in preparation.

[6] D. P. Williams, *Crossed Products of C*-Algebras*, Mathematical Surveys and Monographs no. 134, American Mathematical Society, Providence RI, 2007.