

### Math 246 (9-10am), Quiz 4.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Find the local maxima and minima of the function

$$f(x) = x^3 + 3x^2 - 9x + 2.$$

We find  $f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x-1)(x+3)$   
 Thus the critical points are  $x=1$  and  $x=-3$ .

We find  $f''(x) = 6x + 6$  and compute  $f''(1) = 12$   
 and  $f''(-3) = -12$ . Thus  $x=1$  is a local minimum and  
 $x=-3$  is a local maximum.

Answer:  $x=-3$  is a local maximum,  $x=1$  is a local minimum

2. Find global extrema of the function  $f(x) = x^3 - 2x^2$  on the interval  $[0, 2]$ .

1) We compute  $f$  at endpoints:  $f(0) = 0$ ,  $f(2) = 2^3 - 2 \cdot 2^2 = 0$ .

2) We compute  $f'(x) = 3x^2 - 4x = x(3x - 4)$  and find  
 the critical points:  $x=0$  and  $x = \frac{4}{3}$ .

3) We compute  $f$  at critical points:  $f(0) = 0$ ,  $f(\frac{4}{3}) = (\frac{4}{3})^3 - 2(\frac{4}{3})^2 =$   
 $= (\frac{4}{3})^2(\frac{4}{3} - 2) = \frac{16}{9} \cdot (-\frac{2}{3}) = -\frac{32}{27}$ .

4) Answer: the global maximum is 0, attained at  $x=0$  and  $x=2$ ;  
 the global minimum is  $-\frac{32}{27}$ , attained at  $x = \frac{4}{3}$

3. What is largest value attained by the function  $f(x) = xe^{-2x}$  for positive values of  $x$ ?

We compute  $f'(x) = 1 \cdot e^{-2x} + x \cdot (e^{-2x} \cdot (-2)) = e^{-2x}(1 - 2x)$ .

Thus the only critical point is  $x = \frac{1}{2}$ . Note  
 that  $f'(x) > 0$  for  $x < \frac{1}{2}$  and  $f'(x) < 0$  for  $x > \frac{1}{2}$ ;

so  $x = \frac{1}{2}$  is the point of global maximum.

Thus the largest value of  $f$  for positive  
 values of  $x$  is  $f(\frac{1}{2}) = \frac{1}{2} e^{-2 \cdot \frac{1}{2}} = \frac{1}{2e}$ .

Answer: the largest value of  $f(x)$  is  $\frac{1}{2e}$ ,  
 it is attained at  $x = \frac{1}{2}$ .

Comment: answer will not change if we consider all values  
 of  $x$  positive and negative.