Math 246 (9-10am), Quiz 5.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Find $\lim_{x\to\infty} \frac{2x^2-3x+5\ln(x)}{100x-6x^2+7}$.

It seems that x^2 is the leading term for both numerator and denominator, so

$$\lim_{x\to\infty} \frac{2x^2-3x+5\ln(x)}{100x-6x^2+7} = \lim_{x\to\infty} \frac{(2x^2-3x+5\ln(x))\frac{1}{x^2}}{(100x-6x^2+7)\frac{1}{x^2}} = \lim_{x\to\infty} \frac{2-\frac{3}{x}+5\frac{\ln(x)}{x^2}}{\frac{100}{x}-6+\frac{7}{x^2}} = \frac{2}{-6}$$

(we used that $\ln(x)$ grows slower than x^2).

Answer: the limit is $\frac{2}{-6} = -\frac{1}{3}$.

2. Find $\lim_{x\to 1} \frac{x^3+3x-4}{x^9-2x^7+1}$.

This is indeterminate form of type $\frac{0}{0}$.

Thus we use L'Hospital's Rule:

$$\lim_{x\to 1} \frac{x^3+3x-4}{x^9-2x^7+1} = \lim_{x\to 1} \frac{3x^2+3}{9x^8-2\cdot 7x^6} = \frac{6}{9-14} = -\frac{6}{5}$$

Answer: the limit is $-\frac{6}{5}$.

3. Which of the following functions approaches infinity faster as x approaches infinity: $30x^{1.6}\ln(x)$ or $20\sqrt[3]{x^5}$?

We have $20\sqrt[3]{x^5} = 20x^{5/3} = 20x^{1.666\ldots}$

Since $1.666\ldots > 1.6$ we expect that the second function grows faster. Indeed:

$$\lim_{x\to\infty} \frac{30x^{1.6}\ln(x)}{20\sqrt[3]{x^5}} = \lim_{x\to\infty} \frac{3x^{1.6}\ln(x)}{2x^{1.6666\ldots}} = \lim_{x\to\infty} \frac{3\ln(x)}{2x^{0.0666\ldots}} = 0$$

since any positive power of x grows faster than $\ln(x)$.

Answer: the function $20\sqrt[3]{x^5}$ grows faster than $30x^{1.6}\ln(x)$