

## Math 246, Review problems for the Final Exam.

1. Find the derivative of  $f(t) = \sin(kt^2 + at)$  assuming that  $a$  and  $k$  are constants.
2. Find the derivative of  $f(x) = \frac{\tan(x)+x}{x+1}$ .
3. Find the second derivative of  $f(x) = x^2 \ln(x)$ .
4. Find the tangent line to  $y = e^x + \ln(x+1) - \sin(x)$  at  $x = 0$ .
5. Assume that  $x^3 + xy + y^3 = 4$ . Find  $\frac{dy}{dx}$ .
6. Assume that  $\sin(y) = x + y$ . Find the second derivative of  $y$  with respect to  $x$ .
7. Find the tangent line to the curve  $x^4 + y^3 = x + y$  at the point  $(1, 1)$ .
8. Assume that  $\ln(x + y - 3) = y(x - 1)$ . Find  $\frac{dx}{dt}$  when  $x = 1$  and  $\frac{dy}{dt} = 3$ .
9. For which values of  $x$  the graph of function  $f(x) = 2 \ln(x + 1) + x^2$  is concave up?
10. Consider the discrete time dynamical system  $a_{n+1} = \frac{a_n}{0.5+a_n}$ . Find the equilibria. Which of them are stable?
11. Let  $N_{t+1} = \frac{3}{6-N_t}$  with  $N_0 = 2$ . What is  $N_{100}$  approximately?
12. Find the global maximum and minimum of  $f(x) = x^2 + x + 1$  on the interval  $[-1, 1]$ .
13. Find the critical points of  $f(x) = x^3 - 2x^2 + x - 3$  and determine their types (local maxima or minima).

14. Find the global maximum of  $f(x) = x^2\sqrt{1-x}$  on the interval  $0 \leq x \leq 1$ .

15. A sheet of cardboard 3 ft by 4 ft will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume ?

16. Car B is 30 miles directly east of Car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. What will be the minimum distance between the cars and at what time  $t$  does the minimum distance occur ?

17. Find the limit  $\lim_{x \rightarrow \infty} \frac{3x+2}{2x-3}$  and determine how large should be the values of  $x$  in order for the output to be within 0.01 of the limit.

18. Find the limit  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3}$ .

19. Find the limit  $\lim_{x \rightarrow \infty} \frac{2x^3 \ln(x) - 3x^2 e^{0.1x} + 7e^{x/11}}{2x^2 e^{x/10} + 6x^5 e^{x/20} - 99\sqrt{x} \ln(x)^{10}}$ .

20. Find the limit  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin(x)}$ .