Math 246, Review problems for the Final Exam.

1. Find the derivative of $f(t) = \sin(kt^2 + at)$ assuming that a and k are constants.

Answer: $f'(t) = (2kt + a)\cos(kt^2 + at)$.

2. Find the derivative of $f(x) = \frac{\tan(x)+x}{x+1}$.

Answer: $f'(x) = \frac{(1+\sec^2(x))(x+1)-\tan(x)-x}{(x+1)^2}$.

3. Find the second derivative of $f(x) = x^2 \ln(x)$.

Answer: $f''(x) = 2\ln(x) + 3$.

4. Find the tangent line to $y = e^x + \ln(x+1) - \sin(x)$ at $x = 0$.

Answer: $y = x + 1$.

5. Assume that $x^3 + xy + y^3 = 4$. Find $\frac{dy}{dx}$.

Answer: $\frac{dy}{dx} = -\frac{3x^2+y}{x+3y^2}$.

6. Assume that $\sin(y) = x + y$. Find the second derivative of y with respect to x.

Answer: $y'' = \frac{\sin(y)}{(\cos(y)-1)^3}$.

7. Find the tangent line to the curve $x^4 + y^3 = x + y$ at the point $(1, 1)$.

Answer: $y - 1 = -\frac{3}{2}(x - 1)$.

8. Assume that $\ln(x + y - 3) = y(x - 1)$. Find $\frac{dx}{dt}$ when $x = 1$ and $\frac{dy}{dt} = 3$.

Answer: for $x = 1$ and $\frac{dy}{dt} = 3$ we have $\frac{dx}{dt} = \frac{3}{2}$.

9. For which values of x the graph of function $f(x) = 2\ln(x+1)+x^2$ is concave up?

Answer: $x > 0$ (notice that the function is defined for $x > -1$).

10. Consider the discrete time dynamical system $a_{n+1} = \frac{a_n}{0.5+a_n}$. Find the equilibria. Which of them are stable?

Answer: fixed points are $0, 0.5$. The point 0 is unstable and 0.5 is stable.

11. Let $N_{t+1} = \frac{3}{6-N_t}$ with $N_0 = 2$. What is N_{100} approximately?

Answer: $N_{100} \approx 3 - \sqrt{6}$.

12. Find the global maximum and minimum of $f(x) = x^2+x+1$ on the interval $[-1, 1]$.

Answer: the global maximum is 3 (attained at $x = 1$) and the global minimum is $\frac{3}{4}$ (attained at $x = -\frac{1}{2}$).

13. Find the critical points of $f(x) = x^3 - 2x^2 + x - 3$ and determine their types (local maxima or minima).

Answer: the critical points are 1 and $\frac{1}{3}$; the point 1 is a local minimum and the point $\frac{1}{3}$ is a local maximum.

14. Find the global maximum of $f(x) = x^2\sqrt{1-x}$ on the interval $0 \le x \le 1$.

Answer: the global maximum is $\frac{16}{25\sqrt{5}}$ (attained at $x = \frac{4}{5}$).

15. A sheet of cardboard 3 ft by 4 ft will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume ?

Answer: let $x = \frac{7-\sqrt{13}}{6} \approx .57$. Then the dimensions of the optimal box are $(4-2x) \times (3-2x) \times x$, that is $2.96 \times 1.96 \times .57$.

16. Car B is 30 miles directly east of Car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. What will be the minimum distance between the cars and at what time t does the minimum distance occur ?

Answer: $t = \frac{3}{13} \approx .23 = 13.8$ minutes; the minimum distance is 16.64 miles.

17. Find the limit $\lim_{x\to\infty} \frac{3x+2}{2x-3}$ and determine how large should be the values of x in order for the output to be within 0.01 of the limit.

Answer: $\lim_{x\to\infty} \frac{3x+2}{2x-3} = 1.5$; the values of the output are within 0.01 of 1.5 for $x > 326.5$.

18. Find the limit $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$.

Answer: $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{1}{4}$.

19. Find the limit $\lim_{x\to\infty} \frac{2x^3\ln(x)-3x^2e^{0.1x}+7e^{x/11}}{2x^2e^{x/10}+6x^5e^{x/20}-99\sqrt{x}\ln(x)^{10}}$.

Answer: $\lim_{x\to\infty} \frac{2x^3\ln(x)-3x^2e^{0.1x}+7e^{x/11}}{2x^2e^{x/10}+6x^5e^{x/20}-99\sqrt{x}\ln(x)^{10}} = -\frac{3}{2}$.

20. Find the limit $\lim_{x\to 0} \frac{e^x-1}{\sin(x)}$.

Answer: $\lim_{x\to 0} \frac{e^x-1}{\sin(x)} = 1$.