# Homework 3

## 1. Section 28

28.1 For each of the following functions defined on $\mathbb{R}$, give the set of points at which it is not differentiable. Sketches will be helpful.
(b) $\sin|x|$
(c) $|\sin x|$
(e) $|x^2-1|$

28.2 Use the definition of derivative to calculate the derivatives of the following functions at the indicated points.
(a) $f(x)=x^3$ at $x=2$;
(d) $r(x)=\frac{3x+4}{2x-1}$ at $x=1$.

28.4 Let $f(x)=x^2\sin\frac{1}{x}$ for $x\neq 0$ and $f(0)=0$.
(a) Use Theorems 28.3 and 28.4 to show $f$ is differentiable at each $a\neq 0$ and calculate $f'(a)$. Use, without proof, the fact that $\sin x$ is differentiable and that $\cos x$ is its derivative.
(b) Use the definition to show $f$ is differentiable at $x=0$ and $f'(0)=0$.
(c) Show $f'$ is not continuous at $x=0$.

28.6 Let $f(x)=x\sin\frac{1}{x}$ for $x\neq 0$ and $f(0)=0$. See Fig.19.3.
(a) Observe $f$ is continuous at $x=0$ by Exercise 17.9(c).
(b) Is $f$ differentiable at $x=0$? Justify your answer.

28.10 Let $h(x)=(\cos x+e^x)^{12}$.
(a) Calculate $h'(x)$
(b) Show how the chain rule justifies your computation in part (a) by writing $h=g\circ f$ for suitable $f$ and $g$.

28.16 Let $f$ be a function defined on an open interval $I$ containing $a$. Show $f'(a)$ exists if and only if there is a function $\varepsilon(x)$ defined on $I$ such that

$$f(x)-f(a)=(x-a)[f'(a)-\varepsilon(x)] \quad \text{and} \quad \lim_{x\to a}\varepsilon(x)=0.$$

## 2. Section 29

29.2 Prove $|\cos x-\cos y|\leq|x-y|$ for all $x,y\in\mathbb{R}$.

29.4 Let $f$ and $g$ be differentiable functions on an open interval $I$. Suppose $a,b$ in $I$ satisfy $a<b$ and $f(a)=f(b)=0$. Show $f'(x)+f(x)g'(x)=0$ for some $x\in(a,b)$. *Hint*: Consider $h(x)=f(x)e^{g(x)}$.

## 3. Supplement Homework

1. Exactly one of the following requests is impossible. Decide which it is, and provide examples for the other two. In each case, let's assume the functions are defined on all of $\mathbb{R}$.
(a) Functions $f$ and $g$ not differentiable at zero but where $fg$ is differentiable at zero.
(b) A functions $f$ not differentiable at zero and a functions $g$ differentiable at zero where $fg$ is differentiable at zero.
(c) A functions $f$ not differentiable at zero and a functions $g$ differentiable at zero where $f+g$ is differentiable at zero.