# Homework 9

## 1. Section 34

34.2 Calculate
(a) $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt$ (b) $\lim_{h\to 0} \frac{1}{h}\int_3^{3+h} e^{t^2}\,dt$.

34.3 Let $f$ be defined as follows: $f(t) = 0$ for $t < 0$; $f(t) = t$ for $0 \le t \le 1$; $f(t) = 4$ for $t > 1$.
(a) Determine the function $F(x) = \int_0^x f(t)dt$.
(b) Sketch $F$. Where is $F$ continuous?
(c) Where is $F$ differentiable? Calculate $F'$ at the points of differentiability.

34.6 Let $f$ be a continuous function on $\mathbb{R}$ and define

$$G(x) = \int_0^{\sin x} f(t)dt \qquad \text{for} \quad x \in \mathbb{R}.$$

Show $G$ is differentiable on $\mathbb{R}$ and compute $G'$.

34.11 Suppose $f$ is a continuous function on $[a,b]$. Show that if $\int_a^b f(x)^2dx = 0$, then $f(x) = 0$ for all $x$ in $[a,b]$. *Hint:* See Theorem 33.4.

34.12 Show that if $f$ is a continuous real-valued function on $[a,b]$ satisfying $\int_a^b f(x)g(x)dx = 0$ for every continuous function $g$ on $[a,b]$, then $f(x) = 0$ for all $x$ in $[a,b]$.