1. **Function space**

Consider the set $C$ of continuous functions $f : [0, 1] \to \mathbb{R}$. Show that by suitably defining an addition on $C$, and a multiplication with real numbers, one can make $C$ an additive vector space over $\mathbb{R}$.

(2 points)

2. **Symmetric tensors**

Let $V$ be an $n$-dimensional vector space over $K$ with some basis, let $f : V \times V \to K$ be a bilinear form, and let $t$ be the rank-2 tensor defined by $f$. Show that $f$ is symmetric, i.e., $f(x, y) = f(y, x) \forall x, y \in V$, if and only if the components of the tensor with respect to the given basis are symmetric, i.e., $t_{ij} = t_{ji}$.

(2 points)

3. **The space of rank-2 tensors**

Prove the theorem of ch.1 §1.2: The set of rank-2 tensors forms a vector space of dimension $n^2$ over $\mathbb{R}$.

(3 points)

4. **Cross product of 3-vectors**

Let $x, y \in \mathbb{R}^3$ be vectors, and let $\epsilon_{ijk}$ be the Levi-Civita tensor. Show that the (covariant) components of the cross product $x \times y$ are given by

$$(x \times y)_i = \epsilon_{ijk} x^j y^k$$

(1 points)