16. Energy-momentum tensor
Consider the electromagnetic field in the absence of matter.

a) Show that the tensor field
\[ H_\mu^\nu(x) = (\partial_\mu A_\alpha(x)) \frac{\partial \mathcal{L}}{\partial (\partial_\nu A_\alpha(x))} - \delta_\mu^\nu \mathcal{L} \]
obeyes the continuity equation
\[ \partial_\nu H_\mu^\nu(x) = 0 \quad (\ast) \]

*note:* Notice that \( H_\mu^\nu(x) \) is a generalization of Jacobi’s integral in Classical Mechanics.

b) Show that \((\ast)\) also holds for
\[ \tilde{T}_\mu^\nu = H_\mu^\nu + \partial_\alpha \psi_\mu^{\nu\alpha} \]
where \( \psi_\mu^{\nu\alpha}(x) \) is any tensor field that is antisymmetric in the second and third indices, \( \psi_\mu^{\nu\alpha}(x) = -\psi_\mu^{\alpha\nu}(x) \).

c) Show that \( \psi_\mu^{\nu\alpha} \) can be chosen such that \( \tilde{T}_\mu^\nu(x) = T_\mu^\nu(x) \), which provides an alternative proof that \( T_\mu^\nu(x) \) obeys \((\ast)\).

(5 points)

17. Energy-momentum conservation in the presence of matter
Prove the corollary of ch. 2 § 2.3: In the presence of matter, the energy-momentum tensor obeys the continuity equation
\[ \partial_\nu T_\mu^\nu(x) = -\frac{1}{c} F_\mu^\nu(x) J_\nu(x) \]

(2 points)

18. Energy-momentum tensor for a massive scalar field
Consider the massive scalar field from Problem 11:
\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{m^2}{2} \varphi^2 \]
and the tensor field \( H_\mu^\nu \) defined analogously to Problem 13:
\[ H_\mu^\nu = (\partial_\mu \varphi) \frac{\partial \mathcal{L}}{\partial (\partial_\nu \varphi)} - \delta_\mu^\nu \mathcal{L} \]

Determine \( H_\mu^\nu \) explicitly and show that
\[ \partial_\nu H_\mu^\nu = 0 \]

*hint:* Use the Euler-Lagrange equation determined in Problem 11 a).

(3 points)

.../over
19. **Coulomb gauge**

Consider the 4-vector potential $A^\mu(x) = (\varphi(x), A(x))$. Show that one can always find a gauge transformation such that

$$\nabla \cdot A(x) = 0$$

This choice is called *Coulomb gauge*. 

(2 points)