You should study your midterms and old homework problems, as well as the extra material on this sheet.

1. (a) Define what it means for a sequence \( \{a_n\} \) to converge to \( L \).
(b) Define what it means for a series \( \sum_{n=1}^{\infty} a_n \) to converge to \( L \).
(c) Define the partial sum \( S_n \) for the series \( \sum_{k=1}^{\infty} a_k \).
(d) Define what it means for a series \( \sum a_n \) to be absolutely convergent.
(e) State the Divergence Test, Ratio Test, and Comparison Test.

2. Identify each of the following statements as True, False, or Meaningless:
(a) The sequence \( \{\frac{1}{n}\} \) is divergent
(b) The sequence \( \{\frac{1}{n}\} \) is convergent
(c) The series \( \sum \frac{1}{n} \) is divergent
(d) The series \( \sum \frac{1}{n} \) is convergent
(e) \( \frac{1}{n} \) is divergent
(f) \( \frac{1}{n} \) is convergent
(g) If \( \lim_{n \to \infty} a_n = 3 \) then the sequence is divergent
(h) If \( \lim_{n \to \infty} a_n = 3 \) then the series \( \sum a_n \) is divergent
(i) If \( \lim_{n \to \infty} a_n = 3 \) then it is divergent
(j) If \( \lim_{n \to \infty} a_n = 0 \) then the series \( \sum a_n \) is convergent
(k) If \( \sum a_n \) is convergent then \( \lim a_n = 0 \)
(l) If \( \sum a_n \) converges and \( a_k < b_k \) for all \( k \), then \( \sum b_k \) also converges

3. In each part, give an example for the indicated phenomena:
(a) A series that is convergent but not absolutely convergent
(b) A divergent series \( \sum a_n \) where \( \lim_{n \to \infty} a_n = 0 \).
(c) A convergent series \( \sum a_n \) where \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1 \).
(d) A sequence \( \{a_n\} \) that converges, but where \( \{\frac{1}{a_n}\} \) diverges
(e) A sequence \( \{a_n\} \) that converges, but where \( \sum a_n \) diverges.

4. Does the series \( \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \cdots \) converge or diverge? Explain without using any convergence tests.
5. Does the series \( \sum_{n=1}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n} \right) = \frac{1}{2} - \frac{1}{1} + \frac{1}{3} - \frac{1}{2} + \cdots \) converge or diverge? Explain, and if it converges find the limit.

Answers for #2: FTTFMMFTMFTF