1. Answer each part of this question True or False. If true, give a brief (one or two sentence) explanation. If false, give a counterexample. (You don’t need to give any justification for your counterexample.)

a) For any group $G$, there exists a covering space whose deck transformation group is isomorphic to $G$.

b) If $X$ is a finite CW-complex and $x \in X$, then $\pi_1(X,x)$ is finitely generated.

c) If $X$ is a finite CW-complex and $x \in X$, then $\pi_2(X,x)$ is finitely generated.

2. Let $X$ be the space obtained by attaching two 2-cells to a circle. The first 2-cell is attached via the map $\psi_1 : S^1 \to S^1$ given by $\psi_1(z) = z^4$, and the second 2-cell is attached via the map $\psi_2 : S^1 \to S^1$ given by $\psi_1(z) = z^6$. Compute the fundamental group of $X$. 

3. Let $p : \tilde{X} \to X$ be a covering map, with $X$ and $\tilde{X}$ both path connected, locally path connected, and semilocally simply connected. Let $f : \tilde{X} \to \tilde{X}$ be a map such that $p \circ f = p$. Suppose also that the covering has finitely many sheets. (This hypothesis isn’t necessary, but it makes the problem a little bit easier.) Show that $f$ is a homeomorphism.

4. Draw all of the 3-sheeted covers of $S^1 \vee S^1$, up to isomorphism (hint: there are 7 of them). Which ones are normal? No explanation is required.
5. Compute $\pi_i(\mathbb{C}P^n)$ for all $i \leq 2n + 1$.

6. If $n > 0$, show that the projection $p : S^{2n+1} \to \mathbb{C}P^n$ does not admit a section. (A section is a map $q : \mathbb{C}P^n \to S^{2n+1}$ such that $p \circ q = \text{id}_{\mathbb{C}P^n}$.)

7. Compute $\pi_i(\mathbb{R}P^2 \vee S^2)$ for $i \leq 3$. 
8. Prove that $\pi_{n+k}(S^{n+1} \vee S^{k+1}) \cong \pi_{n+k}(S^{n+1}) \oplus \pi_{n+k}(S^{k+1})$.

9. Show that $\mathbb{R}P^2 \times S^3$ is not homotopy equivalent to $\mathbb{R}P^3 \times S^2$. (Hint: think about both $\pi_1$ and $\pi_2$.)