4. The chain complex is
\[\mathbb{Z} \xrightarrow{\partial_2} \mathbb{Z}^3 \xrightarrow{\partial_1} \mathbb{Z},\]
where \(C_0 \cong \mathbb{Z}\) is spanned by \([v_0] = [v_1] = [v_2]\). We have \(\partial_2[v_0, v_1, v_2] = [v_1, v_2] - [v_0, v_2] + [v_0, v_1]\) and \(\partial_1[v_i, v_j] = [v_i] - [v_j] = 0\).

We have \(H_{i}^\Delta = 0\) for all \(i > 2\) because \(C_i = 0\). We have \(H_2^\Delta = 0\) because \(\partial_2\) is injective. We have \(H_0^\Delta \cong \mathbb{Z}\) because \(\partial_1 = 0\). Finally, \(H_1^\Delta = \mathbb{Z}^3/\text{im}(\partial_2) \cong \mathbb{Z}^2\), since \(\partial_2[v_0, v_1, v_2]\) is primitive.

5. I’ll just give the answer here: \(H_0^\Delta(K) \cong \mathbb{Z}\), \(H_1^\Delta(K) \cong \mathbb{Z} \oplus \mathbb{Z}_2\), and \(H_i^\Delta(K) = 0\) \(\forall i > 1\).

6. The chain complex is
\[\mathbb{Z}^{n+1} \xrightarrow{\partial_2} \mathbb{Z}^{n+1} \xrightarrow{\partial_1} \mathbb{Z},\]
where \(C_2\) has basis \(a_i := [v_0, v_1, v_2]\), for \(0 \leq i \leq n\), \(C_1\) has basis \(b_i := [v_0, v_1]_i\) for \(0 \leq i \leq n\), and \(C_0\) has a single basis element \(c\) given by the unique 0-simplex. The maps are given by \(\partial_2(a_i) = 2b_i - b_{i-1}\) (with the convention that \(b_{-1} = b_0\)) and \(\partial_1 = 0\).

We have \(H_0^\Delta \cong \mathbb{Z}\) because \(\partial_1 = 0\). We have \(H_2^\Delta = 0\) because \(\partial_2\) is injective (the matrix is upper triangular with nonzero entries on the diagonal). We have \(H_1^\Delta = \text{coker}(\partial_2) \cong \mathbb{Z}_2^n\), spanned by \(b_n\).

9. For \(1 \leq i \leq n\), the map
\[\mathbb{Z} \cong C_i \xrightarrow{\partial_i} C_{i-1} \cong \mathbb{Z}\]
is zero if \(i\) is odd and an isomorphism if \(i\) is even. Thus \(H_0^\Delta \cong \mathbb{Z}\), \(H_n^\Delta \cong \mathbb{Z}\) if \(n\) is odd and 0 if \(n\) is even, and all other homology groups are zero.