The principal goals of this class are as follows:

- Introduce interesting classes of examples of symplectic manifolds, including partial flag manifolds, polygon spaces, and toric varieties.
- Prove Delzant’s theorem, which says that the constructive definition of a toric variety (start with a polyhedron and build a space out of it) is equivalent to the abstract definition.
- Use Hamiltonian torus actions to understand Betti numbers (via Morse theory) and cohomology rings (via GKM theory and Kirwan surjectivity) of symplectic manifolds.

Prerequisites: I expect students to be familiar with the definitions and basic properties of smooth manifolds and differential forms (for example, at the level of Chapters I and IV of Guillemin and Pollack’s *Differential Topology*). I also expect you to have some basic familiarity with homology and cohomology of manifolds, at the level of Math 634-6. In particular, you should be comfortable with cohomology rings and with basic functorial operations (pullback and proper pushforward). I will not assume any familiarity with equivariant cohomology or Morse theory, though if you have either one, that’s awesome.

Homework: I have included lots of exercises in my lecture notes (available on my web page), and I will be available to discuss them during my office hours. No homework will be collected.

Plan: My lecture notes are divided into five sections.

- Section 1 begins with a review of differential topology. I expect you to be pretty familiar with the material on differential forms, though I will refresh your memory. The material on Lie groups and Lie algebras may be new to many of you, so I will try to slow down when we reach that point.
- Section 2 gives the basic definitions and examples of symplectic manifolds, culminating in Darboux’s theorem, which says that dimension is the only local invariant of a symplectic manifold.
- Section 3 is about Hamiltonian group actions, which are the main technical tool that we will use for studying symplectic manifolds. In this section we introduce toric varieties, both abstractly and constructively.
- Section 4 begins with a whirlwind review of Morse theory, and then applies it to symplectic manifolds with Hamiltonian torus actions. In particular, we will show how to use a Hamiltonian circle action with isolated fixed points to compute the Betti numbers of a symplectic manifold. The two main theorems in this section are the Atiyah/Guillemin-Sternberg theorem, which characterizes the image of a moment map, and Delzant’s theorem, which relates the abstract and constructive definitions of toric varieties.
- Section 5 is about equivariant cohomology, including Atiyah-Bott localization, GKM theory, and Kirwan surjectivity. We will learn how to compute intersection numbers and equivariant cohomology rings of lots of symplectic manifolds, including toric varieties, partial flag varieties, and polygon spaces.
- If there is any time left, we may discuss geometric invariant theory and the Kirwan-Ness theorem (symplectic quotient = GIT quotient), and illustrate it by relating Horn’s problem about eigenvalues of Hermitian matrices to tensor product multiplicities for representations of GL(n; C). You can read about this material here: http://front.math.ucdavis.edu/9911.5088.