4. It is immediate from the definitions that, for any map \( f : X \to Y \), any path \( h \) from \( y_0 \) to \( y_1 \), and any element \( \delta \in \pi_n(Y, y_0) \), we have \( f_* \circ \beta_h(\delta) = \beta_{f \circ h}(f_* \delta) \). Applying this with \( Y = \tilde{X} \), \( f = p \), \( h = \tilde{\beta} \), and \( \delta = \gamma_*(\alpha) \), we find that
\[
p_* \circ \beta_{\gamma} \circ \gamma_*(\alpha) = \beta_{\gamma} \circ p_* \circ \gamma_*(\alpha) = \beta_{\gamma} \circ p_*(\alpha) =: \gamma \cdot p_*(\alpha),
\]
where the second equality follows from the fact that \( \gamma \) acts by deck transformations, so \( p \circ \gamma = p \).

11. By Whitehead’s theorem, it is sufficient to prove that \( X \) is connected and \( \pi_n(X) \) is trivial for all \( n \geq 1 \). Connectedness is easy: if there were two 0-cells in different connected components of \( X \), they would both need to be contained in \( X_k \) for some finite \( k \), and then \( X_k \) would not be contractible in \( X_{k+1} \). For the second statement, it is sufficient to show that every map from \( S^n \) to \( X \) is nullhomotopic. As in the proof of Theorem 4.8, a map from \( S^n \) to \( X \) can only meet finitely many cells, so we may assume that it lands in some \( X_k \). Then our hypothesis tells us that this map can be homotoped to a constant map inside of \( X_{k+1} \).

12. Let \( X \) be an \( n \)-connected CW complex. Applying Corollary 4.16 when \( A \) is a single 0-cell, we see that there exists a CW complex \( Z \) which is homotopy equivalent to \( X \) and has the property that \( Z \) has one 0-cell and no \( k \)-cells for \( 1 \leq k \leq n \). Let \( f : X \to Z \) be a homotopy equivalence. By cellular approximation, we may assume that the image of \( f \) lies in the \( n \)-skeleton of \( Z \), which is a single point! Hence \( f \) is a nullhomotopic homotopy equivalence, which implies that \( X \) and \( Z \) are contractible.

14. Let \( f : X \to Y \) and \( g : Y \to X \) be homotopy inverses. By cellular approximation, we may assume that both are cellular maps. We claim that the restrictions \( f_n : X_n \to Y_n \) and \( g_n : Y_n \to X_n \) to the \( n \)-skeleta are homotopy inverses. To see this, consider the homotopy from \( g \circ f \) to \( \text{id}_X \), which is a map \( H : X \times [0, 1] \to X \). Again by cellular approximation, we can assume that \( H \) restricts to a map from \( X_n \times [0, 1] \) to \( X_{n+1} = X_n \), which would be a homotopy from \( g_n \circ f_n \) to \( \text{id}_{X_n} \). The argument for \( f_n \circ g_n \) is identical.