1. Cindy works at an hourly job where her pay is determined by a function $P$. If Cindy works an average of $t$ hours a week over the course of a year then she makes $P(t)$ dollars in that year where

$$P(t) = 600t + 800.$$ 

Additionally, the amount that Cindy puts into savings depends on the amount of money that she makes in a year according to a function $S$. That is, if she makes $d$ dollars in a year then she will put $S(d)$ dollars into her savings account that year where

$$S(d) = \frac{3d - 8000}{20}.$$ 

(a) She wants to know how many hours a week that she needs to work in order to make a given amount of money. Find a function $f$ such that if she wants to make $d$ dollars in a year then the average number of hours she needs to work per week during that year is $f(d)$. 

Answer: $f = P^{-1}$ so $f(d) = \frac{d - 800}{600}$

(b) Cindy’s mom is worried about her and wants to know how much she will save depending on how many hours she works. Find a function $g$ such that if she works an average of $t$ hours a week over the course of a year then she puts $g(t)$ dollars into savings in that year.

Answer: $g = S \circ P$ so that $g(t) = 90t - 280$

(c) Cindy also wants to know how many hours a week that she needs to work in order to save a given amount of money. Find a function $h$ such that if she wants to save $m$ dollars in a year then the average number of hours she needs to work per week during that year is $h(m)$.

Answer: This can be done with either $h = g^{-1}$ or $h = P^{-1} \circ S^{-1}$ so that $h(m) = \frac{m + 280}{90}$

2. The number of guitars that a particular store sells in a week is a function of the price for which they sell them. If they sell the guitars for $p$ dollars then they sell $g(p)$ guitars where

$$g(p) = \frac{5(360 - p)}{p - 80}.$$ 

The store’s costs in a week are a function of the number of guitars that they sell that week. If they sell $n$ guitars then their costs for that week are $c(n)$ dollars where

$$c(n) = \frac{(n + 2)^2 + 40000}{500}.$$ 

(a) Find a function $M$ such that if the store wants to sell $n$ guitars in a week then the store needs to charge $M(n)$ dollars for each guitar.

Answer: $M(n) = g^{-1}(n) = \frac{800x + 1800}{x + 5}$

(b) Find a function $f$ such that if the store decides to sell their guitars for $p$ dollars then the store’s costs for the week are $f(p)$ dollars. Be sure to simplify your answer completely.

Answer: $f(p) = \frac{40009p^2 - 6409840p + 2586895600}{500p^2 - 80000p + 3200000}$

3. Alicia works as a waitress in a restaurant. She uses the functions $T$ and $P$ to approximate the money she makes in an evening. If she waits on a table whose total bill is $p$ dollars then she will get a tip of approximately $T(p)$ dollars where

$$T(p) = \frac{3}{20} (p + 10).$$

Also, if she waits on a table seating $n$ people then the total bill at that table is approximately $P(n)$ dollars where

$$P(n) = 15n - 5\sqrt{n} + 10.$$ 

(a) Find a function $f$ such that if she waits on a table seating $n$ people then she gets a tip of approximately $f(n)$ dollars.

Answer: $f(n) = (T \circ P)(n) = \frac{9}{10} n - \frac{3}{2} \sqrt{n} + 3$

(b) Define the function $g$ as follows: If, after a party leaves the table, she sees that they tipped $t$ dollars then the party’s total bill was $g(t)$. Find and simplify a formula for $g(t)$.

Answer: $g(t) = T^{-1}(t) = \frac{20}{3} t - 10$
4. Ned, the park ranger, monitors animal populations in a particular park. He finds that \( m \) months into the year there are approximately \( W(m) \) thousand wolves in the park where

\[
W(m) = e^{-0.01(m-6)^2}.
\]

Additionally, the wolf population influences the rabbit population since wolves feed on rabbits. Ned finds that whenever there are \( w \) thousand wolves in the park there are approximately \( R(w) \) thousand rabbits in the park where

\[
R(w) = 2 - 10 \ln w.
\]

*In this context, \( m = 0 \) corresponds to the first moment of January and \( m = 12 \) corresponds to the last moment of December but this is not really important to the problem.*

(a) At what moment(s) during the year (if any) are there 500 wolves in the park? (Any answers should be in months rounded to two decimal places.) Is \( W \) an invertible function?

*Answer:* There are no months during which there are 500 wolves in the park.

(b) Let \( r \) be the number of rabbits in the park (in thousands) when there are \( r \) thousand rabbits in the park. Find and simplify a formula for \( V(r) \).

*Answer:* \( V(r) = R^{-1}(r) = e^{0.1r - 0.2} \)

(c) Define \( F(m) \) to be the number of rabbits (in thousands) in the park \( m \) months into the year. Find and simplify a formula for \( F(m) \).

*Answer:* \( F(m) = (R \circ W)(m) = 0.1m^2 - 1.2m + 5.6 \)

5. The function \( V(r) = \frac{4}{3} \pi r^3 \) gives the volume (in \( \text{in}^3 \)) of a sphere with radius \( r \) (in inches).

(a) If we are inflating a spherical balloon at a rate such that its radius after \( t \) seconds is \( R(t) = 7 \log(t + 1) \) (in inches) then find a function \( S \) such that after \( t \) seconds the volume of the balloon is \( S(t) \) (in \( \text{in}^3 \)).

*Answer:* \( S(t) = 1372 \pi (\log(t + 1))^3 \)

(b) Define \( R(v) \) to be the radius of a circle whose volume is \( v \). Find and simplify a formula for \( R(v) \).

*Answer:* \( R(v) = V^{-1}(v) = \sqrt[3]{\frac{3v}{4\pi}} \)

6. If a computer manufacturer sells \( u \) computers in a month then his revenue for the month (in dollars) is approximated by \( R(u) = 1000 + 1200u \). If he spends \( d \) dollars on advertising in a month then he will sell \( C(d) = 0.7\sqrt{d} - 3 \) computers in that month.

(a) Does the function \( R \circ C \) have any meaning in the context of this model? If so, explain the meaning and find a formula for the function.

*Answer:* \( R \circ C \) is the function which inputs advertising dollars and outputs revenue. \((R \circ C)(d) = 840\sqrt{d} - 2600 \)

(b) Does the function \( C \circ R \) have any meaning in the context of this model? If so, explain the meaning and find a formula for the function.

*Answer:* \( C \circ R \) has no meaning in the context of this model.

(c) Is \( R \) an invertible function? If so, find \( R^{-1} \) and explain its meaning in the context of the model.

*Answer:* \( R \) is invertible; \( R^{-1} \) inputs revenue and outputs the number of computers sold in the month. \( R^{-1}(r) = \frac{1}{1200} x - \frac{5}{6} \)

(d) Is \( C \) an invertible function? If so, find \( C^{-1} \) and explain its meaning in the context of the model.

*Answer:* \( C \) is invertible; \( C^{-1} \) inputs the number of computers sold and outputs the amount of money (in dollars) that is spent on advertising. \( C^{-1}(u) = \frac{100}{49} u^2 + \frac{600}{49} u + \frac{900}{49} \)