Section 4: Written Homework

Problems: 2, 6, 12, 14, 16, 18, 20, 42, 50, 72

2. \( \ln \left( e^{1/3} \right) = \frac{1}{3} \)

6. \( \log_2 \left( 4^{1/3} \right) = \frac{1}{6} \)

12. \( 3^{h-1} = 14 \Rightarrow (h-1) \log 3 = \log 14 \Rightarrow h-1 = \frac{\log 14}{\log 3} \Rightarrow h = 1 + \frac{\log 14}{\log 3} = 3.402 \)

14. \( (e^n)^2 - e^n = 6 \)

First, let \( u = e^n \)

\[ u^2 - u - 6 = 0 \]

\[ (u - 3)(u + 2) = 0 \]

\[ u = 3 , \ u = -2 \]

- If \( u = 3 \) then \( e^n = 3 \Rightarrow n = \ln 3 = 1.0986 \)
- If \( u = -2 \) then \( e^n = -2 \Rightarrow n = \ln (-2) \), but there is no such \( n \).

\[ n = 1.0986 \]

10. \( \log_2 (4-r) = -3 \Rightarrow 4-r = 2^{-3} \Rightarrow r = 4 - 2^{-3} = 3.875 \)

16. \( \log (n-1) - \log (n+1) = -1 \Rightarrow \log \left( \frac{n-1}{n+1} \right) = -1 \Rightarrow \frac{n-1}{n+1} = 10^{-1} \)

\[ \frac{m+1}{m-1} = \frac{1}{10} \Rightarrow 10m-10 = m+1 \Rightarrow 9m = 11 \Rightarrow m = \frac{11}{9} = 1.2222 \]
20. \( g(t) = 5(0.75)^t \); Suppose \( g(t) = a e^{kt} = a(e^k)^t \). Then \( 0.75 = e^k \Rightarrow k = \ln(0.75) = -0.28776 \).

The continuous growth rate is \( -0.28776 \).

42. \( \ln(r) = 70 + \frac{3}{4} \ln(m) \)
\[ \Rightarrow \ln(r) = 70 + \frac{3}{4} \ln(m) \]
\[ \Rightarrow \frac{4}{3} \ln(r) - \frac{280}{3} = \ln(m) \]
\[ \Rightarrow m = e^{\left(\frac{4}{3} \ln(r) - \frac{280}{3}\right)} \]
\[ \Rightarrow m = e^{\left(\frac{4}{3} \ln(r) - \frac{280}{3}\right)} \]
\[ \Rightarrow m = e^{\left(\frac{4}{3} \ln(r) - \frac{280}{3}\right)} \]

50. If \( P \) is the peak value and \( S(t) \) is the annual sales after \( t \) years, then \( S(t) = P(0.89)^t \). Annual sales fall to 50% of their peak when \( S(t) = 0.05P \).

\[ S(t) = 0.05P \Rightarrow P(0.89)^t = 0.05P \Rightarrow (0.89)^t = 0.05 \Rightarrow t = \frac{\ln(0.05)}{\ln(0.89)} = 25.704 \text{ years} \]

42. \[ 30 = 10 \log \left( \frac{I}{10^{-12}} \right) \Rightarrow 3 = \log \left( \frac{I}{10^{-12}} \right) \Rightarrow 10^3 = \frac{I}{10^{-12}} \Rightarrow I = 10^{-9} \]
\[ 10^{-9} \text{ W/m}^2 \]