1. Let \( f(x) = -0.5x^2 - x + 4 \). (You may find it helpful that \( f(x) = -\frac{1}{2}(x^2 + 2x - 8) \); some people prefer that form.) Round your answers to two decimal places in this problem if necessary.

   (a) (1pt) Find the \( y \)-intercept of \( f \).
   
   Answer: (0, 4)

   (b) (2pt) Find all \( x \)-intercepts of \( f \) (if there are any).
   
   Answer: \((-4, 0) \text{ and } (2, 0)\)

   (c) (2pt) Find the vertex of \( f \).
   
   Answer: \((-1, 4.5)\)

   (d) (5pt) Sketch the graph of \( y = f(x) \) on the axes provided below. This is only a sketch so it doesn’t need to be perfect. However, your sketch does need to include any information that you have found in earlier parts of the problem.

2. A farmer owns a large farm. There is a railroad that runs diagonally across the corner of her farm which creates a small piece of land in the shape of a triangle with the dimensions shown in the diagram below. She wants to build a rectangular garden inside of this triangle but she hasn’t decided on the dimensions of the garden yet. The garden is to have a length of \( x \) meters and a width of \( y \) meters and will fit in the triangle as shown by the shaded area of the diagram. (Note that the corners of the rectangle need to touch the edges of the triangle so as not to waste any space.)

The farmer has hired you to do some analysis of her situation; answer the questions posed below. (Note: You do not need to know any trigonometry to do this problem.)

   (a) (5pt) Find an equation relating the length, \( x \), and the width, \( y \), of the rectangle. Make sure that you solve your equation for \( y \) (so that your answer looks like \( y = \ldots \)).
   
   Answer: \( y = -\frac{3}{5}x + 300 \)
(b) (3pt) Define the function \( A \) so that if \( x \) is the length of her garden then \( A(x) \) is the area of the garden. Find an equation for \( A(x) \).

\[ A(x) = -\frac{3}{5}x^2 + 300 \]

\( \text{Answer: } A(x) = -\frac{3}{5}x^2 + 300 \)

(c) (2pt) What is the practical domain of \( A \)? (Note: A length or width of 0 should not be allowed.)

\( \text{Answer: } (0, 500) \)

BONUS: (2pt) What is the maximum possible area of this rectangular garden? Round to two decimal places if necessary.

\( \text{Answer: } 37500 \text{ m}^2 \)

3. A scientist is studying the effects of pollution on the salmon population near an industrial waste facility. He found that \( t \) years after the facility was opened in 1995, the population of salmon in one particular river was \( P(t) \) salmon where

\[ P(t) = \frac{2500}{2t + 5} \]

He claims that this is an accurate model through the year 2010. (Note: Make sure that you are careful to include units when necessary during this problem.) Round your answers to two decimal places in this problem if necessary.

(a) (1pt) What was the population of salmon in the river in 2005?

\( \text{Answer: } 100 \text{ salmon} \)

(b) (2pt) Find the \( y \)-intercept of \( P \) and interpret its meaning.

\( \text{Answer: } \) The \( y \)-intercept of \( P \) is the point \((0, 500)\). This means that when the facility opened in 1995 there were 500 salmon in the river.

(c) (1pt) There is no \( x \)-intercept of \( P \). (You don’t have to explain why.) What does this tell you about the salmon population in this river?

\( \text{Answer: } \) The salmon in this river don’t die out completely during the scientist’s study.

(d) (2pt) What is the mathematical domain of \( P \)?

\( \text{Answer: } (-\infty, -2.5) \cup (-2.5, \infty) \)

(e) (1pt) What is the practical domain of \( P \)?

\( \text{Answer: } [0, 15] \)

(f) (3pt) Find the average rate of change of \( P \) on the interval \([0, 10]\) and interpret its meaning.

\( \text{Answer: } \) The average rate of change is \(-40 \text{ salmon per year} \). This means that the salmon population in this river dropped by an average of 40 salmon per year from 1995 to 2005

4. Melissa works as a consultant for an engineering firm. Every month she is paid a fixed retainer fee of $5000. She makes additional money depending on the number of hours that she works. For the first 150 hours she works in a month she is paid $100 per hour. If she works more than 150 hours then it is considered overtime; she makes $125 per hour for each hour that she works after the first 150 hours in each month. (Assume that it is possible for her to work a fractional number of hours like 150.2.) Round your answers to two decimal places in this problem if necessary.

(a) (1pt) How much money does she make in a month where she works 120 hours?

\( \text{Answer: } $17000 \)

(b) (2pt) How much money does she make in a month where she works 175 hours?

\( \text{Answer: } $23125 \)

(c) (7pt) Define the function \( M \) such that if she works \( h \) hours in a month then she will make a total of \( M(h) \) dollars in that month. Find an equation for \( M(h) \). (Hint: \( M \) is a piecewise function.)

\( \text{Answer: } M(h) = \begin{cases} 100h + 5000 & \text{if } 0 \leq h \leq 150 \\ 125h + 1250 & \text{if } h > 150 \end{cases} \)