Again, be aware that these “practice problems” don’t cover everything that could possibly be on the exam. I came up with as many examples as I could but it’s not exhaustive by any means.

Here are some things that you need to know how to do:

- Compute end behavior of polynomial, rational, exponential, and logarithmic functions.
- Develop functions based off of exponential models and proportionality.
- Work with compound interest.
- Solve exponential, logarithmic, and rational equations.
- Understand, use, and find the various types of exponential growth rates.
- Understand domain, intercepts, and other graphical features of exponential, logarithmic, and rational function.
- Understand properties of logarithms and exponents.
- Understand the definition of the function $\log_b$.
- Answer questions about exponential and logarithmic models.

Here are some problems:

1. Solve the following equations for the appropriate variable:

   (a) $5 = \frac{x+6}{x-3}$
   
   \textbf{Answer:} $x = \frac{21}{4}$

   (b) $\frac{x^2+3x-10}{x-7} = 0$
   
   \textbf{Answer:} $x = -5, x = 2$

   (c) $\frac{x^2+3x-10}{x-7} = 1$
   
   \textbf{Answer:} $x = 1, x = -3$

   (d) $\frac{3}{r^2} - \frac{5}{r-3} = \frac{r}{r^2-r-6}$
   
   \textbf{Answer:} $x = -\frac{19}{3}$

   (e) $3^{2-x} = -4$
   
   \textbf{Answer:} No solution

   (f) $e^{3t+4} = 22$
   
   \textbf{Answer:} $t = \frac{1}{3} (\ln(22) - 4) \approx -0.303$

   (g) $1 - 4(7^{t^2-1}) = -199$
   
   \textbf{Answer:} $t = \sqrt{\frac{\ln(350)}{\ln(7)}} \approx 1.735, t = -\sqrt{\frac{\ln(350)}{\ln(7)}} \approx -1.735$

   (h) $\log(3s+1) = 4$
   
   \textbf{Answer:} $s = 3333$

   (i) $5 \ln(q + 2) - 4 = 21$
   
   \textbf{Answer:} $q = e^5 - 2 \approx 146.41$

   (j) $\log_3(m - 3) + \log_3(m + 5) = 2$
   
   \textbf{Answer:} $m = 4$

   (k) $\log_3(m - 3) - \log_3(m + 5) = 2$
   
   \textbf{Answer:} No solution
2. For each of the functions below, find the behavior as \( x \) goes to infinity and as \( x \) goes to negative infinity.

(a) \( p(x) = -x^2 + 13x^5 - 4x^3 \)

**Answer:** As \( x \) goes to \( \infty \), \( p(x) \to -\infty \); As \( x \) goes to \(-\infty \), \( p(x) \to -\infty \)

(b) \( p(x) = 0.01x^7 - 7x^4 + 1043x \)

**Answer:** As \( x \) goes to \( \infty \), \( p(x) \to \infty \); As \( x \) goes to \(-\infty \), \( p(x) \to -\infty \)

(c) \( p(x) = 13x^4 - 101x^{13} - 5x^2 \)

**Answer:** As \( x \) goes to \( \infty \), \( p(x) \to \infty \); As \( x \) goes to \(-\infty \), \( p(x) \to -\infty \)

(d) \( p(x) = -7x^{15} + 40x^9 - x^{18} \)

**Answer:** As \( x \) goes to \( \infty \), \( p(x) \to -\infty \); As \( x \) goes to \(-\infty \), \( p(x) \to -\infty \)

(e) \( r(x) = \frac{-3}{x^3} \)

**Answer:** As \( x \) goes to \( \infty \), \( r(x) \to 0 \); As \( x \) goes to \(-\infty \), \( r(x) \to 0 \)

(f) \( r(x) = \frac{-x^3 - 2}{6 - 2x^2 + 4x^4} \)

**Answer:** As \( x \) goes to \( \infty \), \( r(x) \to 0 \); As \( x \) goes to \(-\infty \), \( r(x) \to 0 \)

(g) \( r(x) = \frac{45x^3 + 14x^2 - 7}{7x^4 + 2x - 7} \)

**Answer:** As \( x \) goes to \( \infty \), \( r(x) \to \frac{45}{7} \); As \( x \) goes to \(-\infty \), \( r(x) \to -\frac{45}{7} \)

(h) \( r(x) = \frac{4x^3 + 6x^2 - 8}{12x^2 + x - 1} \)

**Answer:** As \( x \) goes to \( \infty \), \( r(x) \to \infty \); As \( x \) goes to \(-\infty \), \( r(x) \to \infty \)

(i) \( f(x) = 100 \left( \frac{1}{3} \right)^x \)

**Answer:** As \( x \) goes to \( \infty \), \( f(x) \to 0 \); As \( x \) goes to \(-\infty \), \( f(x) \to \infty \)

(j) \( f(x) = 2(1.2)^x \)

**Answer:** As \( x \) goes to \( \infty \), \( f(x) \to \infty \); As \( x \) goes to \(-\infty \), \( f(x) \to 0 \)

(k) \( f(x) = \log_2(x) \)

**Answer:** As \( x \) goes to \( \infty \), \( f(x) \to \infty \); \( f(x) \) is not defined as \( x \) goes to \(-\infty \).

(l) \( f(x) = \log_{0.5}(x) \)

**Answer:** As \( x \) goes to \( \infty \), \( f(x) \to -\infty \); \( f(x) \) is not defined as \( x \) goes to \(-\infty \).

(m) \( f(x) = \ln(x) + e^x + \frac{1}{2} \)

**Answer:** As \( x \) goes to \( \infty \), \( f(x) \to \infty \); \( f(x) \) is not defined as \( x \) goes to \(-\infty \).

3. Write each of the following exponential functions as \( f(x) = a \cdot b^x \) for appropriate values of \( a \) and \( b \).

(a) \( f(x) = \frac{1}{4(e^x)} \)

**Answer:** \( f(x) = \frac{1}{4} \left( \frac{1}{e} \right)^x \); \( a = \frac{1}{4} \) and \( b = \frac{1}{e} \)

(b) \( f(x) = 3(4^{0.5x}) \)

**Answer:** \( f(x) = 3(2)^x \); \( a = 3 \) and \( b = 2 \)

(c) \( f(x) = 6^{2x+1} \)

**Answer:** \( f(x) = 6(36)^x \); \( a = 6 \) and \( b = 36 \)

4. A radioactive material is placed in a beaker and allowed to decay exponentially. Assume that \( t \) is measured in hours, that \( t = 0 \) corresponds to the moment the material is placed in the beaker, and that \( M(t) \) is the mass of the material at time \( t \). In each scenario below, make sure you can do the following:
i. Find a formula for \( M(t) \).
ii. Find the relative growth rate.
iii. Find the continuous rate of decay.
iv. Find the amount of time that it takes before the material decays to a mass of 5 mg.

Note (in case “continuous rate of decay” is unclear): If \( M(t) = 500e^{-0.18t} \) then the “continuous growth rate” is \(-18\%\) which means that it is decaying at a continuous rate of \(18\%\).

(a) The mass decays by \(15\%\) every hour and there was an initial total of 500 mg of material in the beaker.

Answer:
   i. \( M(t) = 500(0.85)^t \)
   ii. \(-0.15\) or \(-15\%\)
   iii. \(-\ln(0.85) \approx 0.1625\) or \(16.25\%\)
   iv. Approximately 28.34 hours

(b) The mass decays by \(15\%\) every three hours and there was an initial total of 500 mg of material in the beaker.

Answer:
   i. \( M(t) = 500(0.85)^{\frac{t}{3}} = 500(\sqrt[3]{0.85})^t \)
   ii. \(\sqrt[3]{0.85} - 1 \approx -0.0527\) or \(-5.27\%\)
   iii. \(-\ln(\sqrt[3]{0.85}) \approx 0.0541\) or \(5.41\%\)
   iv. Approximately 85.01 hours

(c) The mass decays at a continuous hourly rate of \(22\%\) and there was an initial total of 500 mg of material in the beaker.

Answer:
   i. \( M(t) = 500e^{-0.22t} \)
   ii. \(e^{-0.22} - 1 \approx -0.1975\) or \(-19.75\%\)
   iii. \(0.22\) or \(22\%\)
   iv. Approximately 20.93 hours

(d) There was an initial total of 500 mg of material in the beaker and eight hours later there was 100 mg of the material left in the beaker.

Answer:
   i. \( M(t) = 500(\sqrt[8]{0.2})^t \)
   ii. \(\sqrt[8]{0.2} - 1 \approx -0.1822\) or \(-18.22\%\)
   iii. \(-\ln(\sqrt[8]{0.2}) \approx 0.2012\) or \(20.12\%\)
   iv. Approximately 22.89 hours

(e) The mass was 300 mg three hours after it was placed in the beaker and 200 mg ten hours after it was placed in the beaker.

   v. In addition to the four questions posed above, find the initial amount that was placed in the beaker.

Answer:
   i. \( M(t) = 300\left(\frac{2}{3}\right)^{-\frac{t}{3}} \left(\sqrt[3]{\frac{2}{3}}\right)^t \), or \( M(t) \approx 356.93(0.9437)^t \)
   ii. \(\left(\sqrt[3]{\frac{2}{3}}\right) - 1 \approx -0.0563\) or \(-5.63\%\)
iii. \(-\ln \left(\frac{7}{\sqrt[3]{2}}\right) \approx 0.0579 \text{ or } 5.79\%\)

iv. Approximately 73.69 hours

v. Approximately 356.93 mg

5. An investment is made in an account. Assume that \(t\) is measured in years, that \(t = 0\) corresponds to the moment that the investment was made, and that \(A(t)\) is the amount of money in the investment at time \(t\). In each of the scenarios below, find a formula for \(A(t)\) (except in part (d)) and then answer the indicated question

(a) The initial investment is $5000 and interest is compounded at a continuous annual rate of 4%. What is the account worth after 30 years?

\[ A(t) = 5000e^{0.04t}; \] $16 600.58

(b) The initial investment is $5000 and interest is compounded monthly at an annual rate of 4%. How long will it take for the account to be worth $6000?

\[ A(t) = 5000\left(1 + \frac{0.04}{12}\right)^{12t} = 5000\left(1.003333\right)^{12t}; 4.57\] years

(c) Interest is compounded continuously at an annual rate of 5% and the investor wants the account to be worth at least $50 000 after 30 years. How much must she invest initially?

\[ A(t) = 11568.88\left(1.05\right)^t; \]$11 568.88

(d) The investor wants his account to grow by 50% after 25 years. What interest rate must he get if...

i. ...interest is compounded yearly?

\[ A(t) = 11568.88\left(1.05\right)^t; \]$11 568.88

ii. ...interest is compounded continuously?

\[ A(t) = 11568.88\left(1.05\right)^t; \]$11 568.88

6. Atmospheric pressure at sea level is approximately 101200 Pascals. For every 1000 m increase in altitude, atmospheric pressure decreases by approximately 12%. Find a function, \(P\), such that the pressure at an altitude of \(h\) meters is \(P(h)\) Pascals.

\[ P(h) = 101200\left(0.88\right)^{0.001h}\]

7. Stuart bought a painting for $5000 in 1990. He had it appraised for $7000 in 2005. Let \(V(t)\) be the value of the painting \(t\) years after he bought it.

(a) Assume that the painting’s value grows linearly.

i. Find an equation for \(V(t)\).

\[ V(t) = \frac{400}{3}t + 5000\]

ii. Interpret the meaning of the slope.

\[ \text{Answer: Every year the painting’s value increases by } \frac{400}{3} \approx 133.33 \text{ dollars.}\]

iii. How long will it take for the painting to be worth $10 000?

\[ \text{Answer: } 37.5 \text{ years}\]

(b) Assume that the painting’s value grows exponentially.

i. Find an equation for \(V(t)\).

\[ V(t) = 5000\left(1.4\right)^t\]

ii. Find and interpret the relative growth rate.

\[ \text{Answer: The relative growth rate is approximately } 2.27\%, \text{ which means that the painting’s value increases by } 2.27\% \text{ every year.}\]
iii. How long will it take for the painting to be worth $10,000?

Answer: Approximately 30.9 years

8. A biologist introduces a population of fish to a small pond. The population initially grows quickly but, due to overcrowding, slows over time. The biologist uses the function $F$ to model the population of the fish in the pond as follows: after $t$ days there are

$$F(t) = 100 + 150 \ln(1 + x)$$

fish in the pond.

(a) How many fish did the biologist introduce to the pond?

Answer: 100

(b) The biologist predicts that the model will be accurate until the fish population reaches 1000. How long does that take?

Answer: Approximately 402.43 days