1. Find the group $\text{Aut}(\mathbb{Z}_8)$.

By a homework problem, $G = \text{Aut}(\mathbb{Z}_8) \simeq \mathbb{Z}_8^*$, so $|G| = 4$. Since for any $g \in \mathbb{Z}_8^* = \{1, 3, 5, 7\}$ we have $g^2 = 1$, the group $G$ is isomorphic to the Klein group $V_4 = \mathbb{Z}_2 \times \mathbb{Z}_2$.

2. Decide if the following statements are True or False and provide a brief, but to the point, justification.

(a) If $f \in \text{Hom}_C(X, Y)$ and $g \in \text{Hom}_C(Y, X)$ are such that $g \circ f = \text{id}_X$, then $f$ is an isomorphism.

False. Take for example $C = \text{Sets}$ and $X = \{1\}$, $Y = \{1, 2\}$, $f(1) = 1, g(y) = 1$.

(b) If $H \trianglelefteq G$ and $K \leq G$, then $HK \leq G$.

True. For $g_1 = h_1k_1$ and $g_2 = h_2k_2$ in $HK$ we have $g_1g_2 = h_1k_1h_2k_2 = h_1(k_1h_2k_1^{-1})k_1k_2 \in HK$, because by normality $k_1h_2k_1^{-1} \in H$. Similarly, $g^{-1} = k_1^{-1}h_1^{-1} = (k_1^{-1}h_1^{-1}k_1)k_1^{-1} \in HK$. So $HK \leq G$.

(c) If $G$ is an abelian simple group, then $|G|$ is prime.

True. If $g \in G$ is a non-identity element, then $\langle g \rangle$ is a non-trivial normal subgroup, so $\langle g \rangle = G$, because $G$ is simple. If $n := |G| = |\langle g \rangle| = |g|$ is not prime, i.e. $n = ab$ for $1 < a, b < n$, then $\langle g^a \rangle$ is a proper normal subgroup of $G$, so $n$ must be prime.