1. (T) Let $G$ be the group of rotations of the cube (i.e. $G \simeq S_4$) and let $V$ be the permutation representation corresponding to the action of $G$ on the vertices of the cube.

   (a) Decompose $V$ into irreducibles.

   (b) Let $T : V \to V$ be transformation that replaces the number at a vertex by the average of the numbers at the three adjacent vertices. Let $w = (w_1, \ldots, w_8) \in V$. Find $\lim_{n \to \infty} T^w w$.

2. Perform a similar analysis for the averaging operator on the faces of a dodecahedron.
   [Hint. It may be helpful (but not necessary) to recall that the group of symmetries of the dodecahedron is isomorphic to $A_5$.]

3. (T) Use the character table of $S_4$ to find the tensor product multiplicities for irreducible representations $L_i$, i.e. compute $N^k_{ij}$ such that $L_i \otimes L_j \simeq \sum_k N^k_{ij} L_k$.

4. (T) Exercise 22.2.25

5. (T) Exercise 22.2.26

6. (T) Exercise 22.2.27

7. Exercise 22.3.7

8. (T) Exercise 22.3.8