1. How many connected 4-fold coverings (up to isomorphism in $\text{Cov}(X)$) the space $X = S^1 \times S^1$ has?

2. (a) How many 2-fold coverings (up to isomorphism in $\text{Cov}(X)$) the space $X = S^1 \lor S^1 \lor S^1$ has?
   
   (b) For each connected covering from part (a) describe the corresponding index 2 subgroup of the free group $\pi_1(X,x_0) \xrightarrow{\sim} \mathbb{Z} \ast \mathbb{Z} \ast \mathbb{Z}$ by specifying generators and relations.

3. (a) How many maps up to homotopy equivalence from $S^n$ to $S^1$ are there? (Consider separately the case $n = 1$.)
   
   (b) How many maps up to homotopy equivalence from $\mathbb{R}P^n$ to $S^1$ are there?

4. Compute the fundamental group of $\mathbb{R}P^n$ with one point removed. (The answer may depend on $n$.)

5. Fix relatively prime positive integers $k$ and $n$. Consider the action of $\mathbb{Z}/n\mathbb{Z}$ on $S^3 = \{(z_1, z_2) \in \mathbb{C}^4 | |z_1|^2 + |z_2|^2 = 1\}$ defined by the formula $m(z_1, z_2) = (\exp(\frac{2\pi i m}{n})z_1, \exp(\frac{2\pi i m k}{n})z_2)$, where $m$ is the class an integer $m$ in $\mathbb{Z}/n\mathbb{Z}$. The lens space $L(n,k)$ is the quotient of $S^3$ modulo this action of $\mathbb{Z}/n\mathbb{Z}$ i.e., $L(n,k) = S^3/\sim$, where $(z_1, z_2) \sim (z'_1, z'_2)$ if there is $m \in \mathbb{Z}/n\mathbb{Z}$ such that $(z_1, z_2) = m(z'_1, z'_2)$. We endow $L(n,k)$ with the quotient topology. Compute the fundamental group of $L(n,k)$. Show that if $L(n,k)$ is homeomorphic to $L(n',k')$ then $n = n'$.

6. Let $X$ be the the complement to 2 skew lines in $\mathbb{R}^3$. Compute the fundamental group of $X$.

7. Let $X = \{(z_1, z_2) \in \mathbb{C}^2 | z_1 \neq z_2\}$ and $Y$ is obtained from $X$ by identifying points $(z_1, z_2)$ and $(z_2, z_1)$. Compute the fundamental group of $Y$.

8. (a) Let $\mathcal{C}$ be a category and $\text{Id}_\mathcal{C} : \mathcal{C} \rightarrow \mathcal{C}$ the identity functor. Show that the group $\text{Aut}_{\text{Funct}(\mathcal{C} \to \mathcal{C})}(\text{Id}_\mathcal{C})$ is commutative.
   
   (b) Let $G$ be a group $\mathcal{C} = G - \text{Sets}$. Show that $\text{Aut}_{\text{Funct}(\mathcal{C} \to \mathcal{C})}(\text{Id}_\mathcal{C})$ is isomorphic to the center of $G$. 

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