Worksheet 10

Wednesday, November 13, 2019

Math 205

1. (a) Argue that the set of finite strings of As and Bs is countable.

   (b) Argue that the set of arbitrary (finite or infinite) strings of As and Bs is uncountable: that is, if you're given any list $x_0, x_1, x_2, \ldots$ of such strings, you can cook up a string y that's not on the list.

2. (a) Argue that the set of terminating decimals between 0 and 1 is countable.

   (b) Argue that the set of real numbers between 0 and 1 is uncountable: that is, if you're given any list $x_0, x_1, x_2, \dots$ of such numbers, you can cook up a number y that's not on the list.

3. (From Monday) Last week we saw that between any two real numbers $a < b$ there is at least one rational number, and at least one irrational number. So maybe you could hope to pair them off. But this week we've seen that there are uncountably many irrational numbers in that interval, and only countably many rational numbers. Is this a paradox, or is it ok?

4. (a) Recall that

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

is the set of natural numbers. Argue that the set of 2-element subsets of $\mathbb{N}$ is countable.

(b) Argue that the finite subsets of $\mathbb{N}$ is countable.

(c) Argue that the set of arbitrary (finite or infinite) subsets of $\mathbb{N}$ is uncountable.

5. (a) Argue that the set of points on the circle $x^2 + y^2 = 1$ is uncountable.

(b) But the set of rational points on the circle $x^2 + y^2 = 1$ — that is, both x and y are both rational numbers — is countable.

6. Still optional:

   (a) Argue that the set of quadratic polynomials $ax^2 + bx + c$, where the coefficients a, b, c are integers, is countable.

   (b) Argue that the set of algebraic numbers (defined on Monday's worksheet) is countable.