

# Worksheet 5

Monday, October 21, 2019

Math 205

1. You have been assigned into groups of three for the week. Write down your colleagues' names and email addresses.

2. As a group, we considered the sequence of points

$$(1, 1) \quad (2, 2) \quad (3, 3) \quad \dots \quad (n, n) \quad \dots$$

in  $\mathbb{R}^2$ , which goes off to infinity. Get a blank sheet of paper and sketch this sequence in the plane.

In homogeneous coordinates, the sequence became

$$(1 : 1 : 1) \quad (2 : 2 : 1) \quad (3 : 3 : 1) \quad \dots \quad (n : n : 1) \quad \dots,$$

which is equivalent to

$$(1 : 1 : 1) \quad (1 : 1 : \frac{1}{2}) \quad (1 : 1 : \frac{1}{3}) \quad \dots \quad (1 : 1 : \frac{1}{n}) \quad \dots,$$

which converges to

$$(1 : 1 : 0).$$

3. For each of the following sequences, add it to your sketch of the plane, and find its limit in the same way as above.

(a)  $(1, 2), (2, 3), (3, 4), \dots, (n, n + 1), \dots$

(b)  $(1, 3), (2, 4), (3, 5), \dots, (n, n + 2), \dots$

(c)  $(1, 2), (2, 4), (3, 6), \dots, (n, 2n), \dots$

(d) Find another sequence that converges to the same limit as (c).

(e)  $(1, 1), (1, 2), (1, 3), \dots, (1, n), \dots$

(f) Find another sequence that converges to the same limit as (e).

(g)  $(1, 1), (2, 4), (3, 9), \dots, (n, n^2), \dots$

4. The original sequence of points  $(x, y)$  in problem 2 satisfies  $y = x$ . When we put it into homogeneous coordinates  $(x : y : z)$ , the same equation still holds, even for the second, rescaled sequence of homogeneous coordinates, and for the limit  $(1 : 1 : 0)$ .

The original sequence in problem 3(a) satisfies  $y = x + 1$ , but when you go into homogeneous coordinates and rescale then this is no longer true, nor is it true in the limit. Can you find an equation involving  $x$ ,  $y$ , and  $z$  that is true at all those points?

Similarly, for each of the remaining parts of problem (3), find an equation in  $x$ ,  $y$ , and  $z$  that works.