

Homework 4

Due Friday, October 22, 2021

Math 205

1. Type in the function `is_prime` that we discussed together, and improve it if you want. Test it on a few examples.
2. How many primes are less than 100? Less than 1,000? Less than 10,000? Can your code deal with 100,000, or is it too slow?
3. What is the 1,000th prime? Here we're counting 2 as the first prime, 3 as the second prime, and so on.
4. Prime numbers of the form $2^n - 1$ are called *Mersenne primes*. It is conjectured that there are infinitely many Mersenne primes. Find all the Mersenne primes less than 1 million.
5. Pairs of prime numbers that are two apart are called *twin primes*: for example 11 and 13, or 17 and 19, or 29 and 31. It is conjectured that there are infinitely many twin primes. Find all the twin primes less than 10,000.
6. Write a function `smallest_prime_divisor` that take an integer n and returns its smallest prime divisor. Test it on a few examples. Then use it to find all the prime factors of 600851475143.
7. Use your function to investigate the smallest prime divisors of numbers of the form $2^n + 1$. Write a program that prints n , $2^n + 1$, and the smallest prime divisor of $2^n + 1$, for n from 1 to 39. Do you see any patterns in your list? Try to precisely and mathematically state at least three patterns that you find. Then see if your patterns continue for bigger values of n .

8. Apart from 2 and 5, every prime has either a 1, 3, 7, or 9 in the last digit of its base-10 representation. We may wonder whether these four possibilities are evenly distributed, or whether one occur more frequently than another. Let's investigate.

First, what does the following code do?

```
count3 = 0
for i in range(1,100):
    if is_prime(i) and i % 10 == 3:
        count3 += 1
print(count3)
```

Next, edit the code above and use it to fill in the following table:

last digit:	1	3	7	9
# of primes < 100 :				
# of primes < 1000 :				
# of primes < 10000 :				

Challenge question: What about other bases than base 10?

9. If you have spare time at the end of the day on Wednesday, think about ways to improve your `is_prime` function, and explore how far you can push all these experiments before they become too slow.