Homework 7

Due Friday, November 12, 2021

Math 205

1. Together we discussed the birthday problem, assuming for convenience that people's birth dates are uniformly distributed through the year, and ignoring leap years. So if there are two people in a room, the probability that they have the same birthday is 1/365, and if there are 366 people in the room, the the probability that at least two of them have the same birthday is 100%.

   (a) Write a program that does the following, for each n from 2 to 366. First, compute the probability that in a room of n people, all of them have different birthdays. Then compute the probability that at least two of them have the same birthday. Then print both probabilities.

   (I'm not asking you to run a simulation, just to use the computer to multiply whatever fractions you need to multiply, because it would be too cumbersome on a calculator.)

   (b) At what point does the probability of having at least two people with the same birthday surpass 50%?

   (c) Optional challenge problem: write a program simulate the following experiment 10,000 times and report the result. Let n be your answer to part (b). Randomly choose n numbers between 1 and 365. Decide whether they were all different, or whether at least two were the same. How does the probability compare to 50%?

   Also try it with other values of n, and see how the result compares to the probabilities you computed in part (a).

2. You randomly choose a point (x, y) in the unit square, so x and y both lie in the unit interval $[0, 1]$.

   (a) Calculate the probability that the distance from (x, y) to the origin is less than or equal to 1. You'll want to draw a picture!

   (b) Write a program to simulate the experiment 10,000 times and report the result. To get a random floating point number between 0 and 1, use `uniform(0,1)` or just `random()`.

   (c) If your calculation and your program don't agree, go back and correct the calculation, or the program, or both.

3. You randomly choose two numbers x and y in $[0, 1]$.

   (a) Calculate the probability that the distance between x and y is greater than or equal to $1/2$.

   (b) Write a program to simulate the experiment 10,000 times and report the result. If your calculation and your program don't agree, go back and correct the calculation, or the program, or both.

   (c) While you're at it, write a program to find the average distance between x and y. Can you guess what the true number is? Can you compute it by hand?

4. Optional challenge problem: repeat #3 for two points (x, y) and (z, w) in the unit square. Do the computer simulations first, then see if the calculations by hand are even feasible.