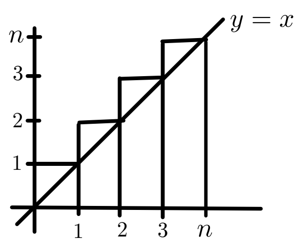
Worksheet 2

Math 206

Monday, October 2, 2023

Write down your group members' names and email addresses. Do they have a favorite number?

Last Wednesday we discussed many ways of thinking about the sum $1 + 2 + \cdots + n$. One way involved this diagram:



The areas of the first vertical bar is 1, the second is 2, and so on, so their total area is the sum we were interested in. Below the diagonal line, we have a big triangle whose area is $n^2/2$. Above the diagonal line we have n small triangles, each with an area of $1/2$. So the total area is

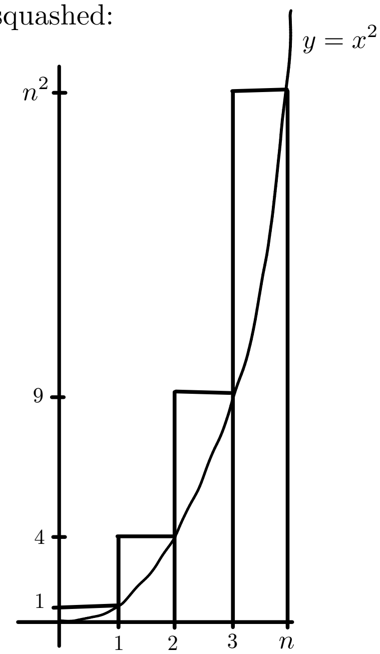
$$\frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}.$$

If you have any lingering questions about this explanation, discuss them with your group.

This week we want to understand $1^2 + 2^2 + \cdots + n^2$. Start by computing the values for n from 1 to 5:

n	$1^2 + 2^2 + \cdots + n^2$
1	
2	
3	
4	
5	

In analogy with the diagram above, consider this diagram, whose vertical scale is somewhat squashed:



The area of the first vertical bar is 1, the second is 4, the third is 9, and so on, so their total area is the sum we're interested in. Does this make sense?

We want to find this total area by computing the area of the big "curvy triangle" below the parabola, and then the areas of the n small curvy triangles above the parabola, although those small curvy triangles aren't all the same size any more.

What is the area of the big curvy triangle below the parabola? You'll need a little calculus to answer this.

Looking at the i^{th} bar, what is the area of the part of the bar that lies below the parabola, as a function of i? Again you'll need some calculus. If you want you can warm up with the first, second, and third bars.

What is the area of the curvy triangle in the i^{th} bar, that is, the part of the bar that lies above the parabola?

Can you add up the areas of the curvy triangles as i goes from 1 to n?

Put all this together to get a formula for $1^2 + 2^2 + \cdots + n^2$. Does it agree with the values you computed earlier?