

Midterm Exam 2

November 23, 2015

Use your own notebook paper. Start each problem on a new sheet of paper. I recommend that you use pencil rather than pen. No calculators, notes, or cheating.

1. Solids of revolution (30 points).

- (a) Let a be a positive number. Sketch the region in the xy -plane that lies above the x -axis and below the parabola $y = a - x^2$. Indicate the coordinates of the points where the parabola meets the x - and y -axes. Separately, sketch the solid obtained by revolving this region around the y -axis. (10 points)
- (b) Find the volume of the solid using the shell method. Your answer will be a function of a . (10 points)
- (c) Find the volume of the solid using the disc method. (10 points)

Your answers to (b) and (c) should agree of course. Moreover, if you plug in $a = 4$ you should get 8π , as we saw on Quiz 5.

2. Center of mass (30 points).

- (a) Sketch the part of the unit disc that lies in the first quadrant. What is its area? Hint: Don't use calculus. (10 points)
- (b) Set up an integral to find either the x - or the y -coordinate of center of mass of this quarter-disc, assuming it has constant density. (By the symmetry of the region, the x - and y -coordinates of the center of mass are the same, so you only need to find one or the other.) (10 points)
- (c) Evaluate the integral you set up in part (b). (10 points)

As a sanity check, you might want to make sure that your point lies inside the region, and in particular that the distance to the origin is less than 1.

3. Hydrostatic force (40 points). A cylindrical drum, 3 feet in diameter, lies on its side, half-full of water. We are interested in the the hydrostatic force on one end of the drum. You will need to know that the pressure underwater equals the depth in feet times 0.43 pounds per square inch. Hint: I recommend working in inches rather than feet; one foot equals 12 inches.

If you prefer to use SI units, say that the drum is 1 meter in diameter, and recall that the pressure underwater is 9800 Pa per meter of depth.

- (a) (10 points) Draw a picture.
- (b) (30 points) Set up the integral, but do not evaluate it.