Practice Midterm 2

November 16, 2015

Use your own notebook paper. Start each problem on a new sheet of paper. I recommend that you use pencil rather than pen. Problem #3 requires a calculator, but the real exam will not.

1. Let a be a positive number.

(a) Sketch the region in the first quadrant bounded by the line $x = a$ and the parabola $y = x^2$. Separately, sketch the solid obtained by revolving this region around the line $x = a$.

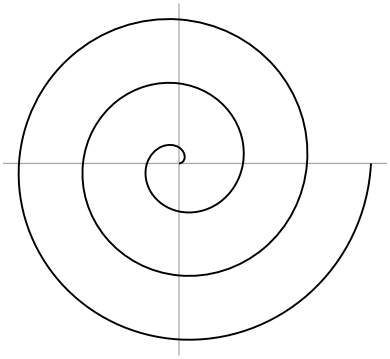
(b) Find the volume of the solid using the shell method. Your answer will be a function of a.

(c) Find the volume of the solid using the disc method.

2. In §6.4 #6 we encountered the parametric curve

$$x = t\cos t \qquad y = t\sin t,$$

called the *spiral of Archimedes*. It looks like this:



(a) Find $(dx/dt)^2$ and $(dy/dt)^2$. Simplify the expression $(dx/dt)^2 + (dy/dt)^2$. Hint: All the sines and cosines go away by the end.

(b) Find the length of the curve traced out as t goes from 0 to π. You will want to refer to #21 on the attached table of integrals; we did a similar integral by hand in lecture when we were finding the arc length of a parabola, but it was long.

3. You sit on the roof of a two-story house, which is 20 feet tall, holding a rope that is 30 feet long and weighs 3 pounds. Your friend on the ground holds a slinky, which we will model as a spring obeying Hooke's law with a natural length of 6 inches and a spring constant of 0.2 pounds per foot; somewhat unrealistically, we will neglect the weight of the spring. You throw one end of the rope down to your friend, who ties it to one end of the slinky and attaches the other end of the slinky to the ground. How much work will you do in pulling the rope back up to the roof and thereby stretching the slinky to the height of the house?

You will need a calculator to punch in the numbers at the end. This will not be the case on the real exam.

If you prefer SI units, make the house 6 m tall, the length of the rope 10 m, the mass of the rope 1.5 kg, the natural length of the slinky 15 cm, and the spring constant 3 N/m. Recall that the acceleration due to gravity is 9.8 m/s^2. Find the work in Newton-meters, i.e. in Joules.

TABLE OF INTEGRALS

Basic Forms

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \dfrac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int a^u\,du = \dfrac{a^u}{\ln a} + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = \ln|\sec u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
16. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\dfrac{u}{a} + C, \quad a > 0$
17. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$
18. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{u + a}{u - a}\right| + C$
20. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u - a}{u + a}\right| + C$

Forms Involving $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2}\,du = \dfrac{u}{2}\sqrt{a^2 + u^2} + \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
22. $\int u^2\sqrt{a^2 + u^2}\,du = \dfrac{u}{8}(a^2 + 2u^2)\sqrt{a^2 + u^2} - \dfrac{a^4}{8}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
23. $\int \dfrac{\sqrt{a^2 + u^2}}{u}\,du = \sqrt{a^2 + u^2} - a\ln\left|\dfrac{a + \sqrt{a^2 + u^2}}{u}\right| + C$
24. $\int \dfrac{\sqrt{a^2 + u^2}}{u^2}\,du = -\dfrac{\sqrt{a^2 + u^2}}{u} + \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
25. $\int \dfrac{du}{\sqrt{a^2 + u^2}} = \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
26. $\int \dfrac{u^2\,du}{\sqrt{a^2 + u^2}} = \dfrac{u}{2}\sqrt{a^2 + u^2} - \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
27. $\int \dfrac{du}{u\sqrt{a^2 + u^2}} = -\dfrac{1}{a}\ln\left|\dfrac{\sqrt{a^2 + u^2} + a}{u}\right| + C$
28. $\int \dfrac{du}{u^2\sqrt{a^2 + u^2}} = -\dfrac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29. $\int \dfrac{du}{(a^2 + u^2)^{3/2}} = \dfrac{u}{a^2\sqrt{a^2 + u^2}} + C$