Final Exam

Name ____

No notes or cheating. You may use a scientific calculator but not a graphing calculator. Each problem is worth 10 points.

1. §6.6 #11: A heavy rope, 30 ft long, weights 0.6 lb/ft and hangs over the edge of a building 70 ft high. How much work is done in pulling the rope to the top of the building?

   (Recall that work = force $\times$ distance.)

   (a) Write an integral to calculate the work.

   (b) Evaluate the integral.

Now suppose that after hauling up half the rope, you stop, leaving 15 feet of rope hanging over the edge of the building. How much work have you done?

(c) Write an integral to calculate the work.

(d) Evaluate the integral.

2. §6.6 #6: A spring has a natural length of 15 cm. If a force of 25 N is required to keep it stretched to a length of 25 cm, how much work is required to stretch it from 15 cm to 20 cm?

   (According to Hooke's law, the force required to hold a stretched spring is proportional to how far it has been stretched beyond its natural length; that is, proportional to the stretched length minus the natural length.)

   (a) Write an integral to calculate the work.

   (b) Evaluate the integral.

3. §6.6 #37: A trough is filled with a liquid of density 1030 kg/m^3. The ends of the trough are equilateral triangles with sides 2 m long and with the vertex at the bottom.

   (a) Draw a rough picture of the trough. How deep is it?

   (b) Draw a triangular cross-section of the trough, and a horizontal line at a height y above the bottom of the trough. How wide is the line? What is the distance from the top of the trough down to the line? (Both answers should be functions of y.)

   (c) Write an integral to calculate the hydrostatic force on one end of the trough. *Do not evaluate the integral.* (Recall that pressure = density $\times$ 9.8 $\times$ depth, that force = pressure $\times$ area, and that the area of this horizontal slice is its width times dy.)

4. §7.3 #15: When a cold drink is taken from a refrigerator, its temperature is 40°F. After 25 minutes in a 70°F room, its temperature has increased to 50°F.

   (According to Newton's law of cooling, the rate at which the drink warms up is proportional to the difference between the temperature of the room and the temperature of the drink.)

   (a) What is the temperature of the drink after 50 minutes?

(b) When will the temperature of the drink be $60°F$?

5. §7.2 #45: The volume of a room is 6000 ft^3. The air in the room contains 0.15% carbon dioxide initially. Fresher air containing only 0.05% carbon dioxide flows in through a window at a rate of 60 ft^3/min. Well-mixed air flows out another window at the same rate, 60 ft^3/min.

   (a) Let y be the amount of carbon dioxide in the room in ft^3, and let t be the time in minutes. Explain why $y = 9$ when $t = 0$.

   (b) How many ft^3/min of carbon dioxide are entering the room? (This is a number.)

   (c) How many ft^3/min of carbon dioxide are leaving the room? (This depends on y.)

   (d) Write a differential equation ($\frac{dy}{dt} = \cdots$) that describes how the amount of carbon dioxide in the room is changing.

(e) What is the percentage of carbon dioxide in the room after an hour?