Midterm 2

Name _____

You may use a page of notes. Each part is worth 3 points.

1. (a) Based on §6.1 #8. Sketch the region bounded the line $y = x + 4$ and the parabola $y = x^2 - 2x$.

   (b) Find the two points where the line and the parabola intersect.

(c) Set up an integral to find the area of the region. *Do not evaluate the integral.*

(d) The perimeter of the region consists of a line segment and a segment of a parabola. The length of the line segment is $5\sqrt{2}$: this doesn't require any calculus. Set up the integral to find the length of the parabolic segment. *Do not evaluate the integral.*

(e) If you had evaluated the integral from part (c), you would have gotten 125/6. Set up an integral to find the x-coordinate of the center of mass of the region. *Do not evaluate the integral.*

(f) Extra credit: Set up an integral to find the y-coordinate of the center of mass of the region. *Do not evaluate the integral.* Hint: If $y = x^2 - 2x$ then $x = 1 \pm \sqrt{y+1}$.

2. On the board you see a sketch of the region that lies above the line $y = 1$ and inside the circle $x^2 + y^2 = 2$.

   (a) Roughly sketch the solid obtained by revolving the region around the x-axis.

   (b) Set up the integral to find the volume of the solid from part (a) using disks/washers. Hint: The top half of the circle can be described as $y = \sqrt{2 - x^2}$.

   (c) Evaluate the integral from part (b).

(d) Set up the integral to find the volume of the solid from part (a) using cylindrical shells. Hint: The right half of the circle can be described as $x = \sqrt{2 - y^2}$.

(e) Evaluate the integral in part (d). This should agree with your answer to part (c). Hint: Substitute $u = 2 - y^2$.

(f) Roughly sketch the solid obtained by revolving the region around the y-axis.

(g) Set up the integral to find the volume of the solid from part (f) solid using cylindrical shells. Hint: The top half of the circle can be described as $y = \sqrt{2 - x^2}$.

(h) Evaluate the integral from part (g). Hints: Substitute $u = 2 - x^2$. Recall that $2^{3/2} = 2^1 \cdot 2^{1/2} = 2\sqrt{2}$.

(i) Set up the integral to find the volume of the solid from part (f) using disks. Hint: The right half of the circle can be described as $x = \sqrt{2 - y^2}$.

(j) Evaluate the integral in part (i). This should agree with your answer to part (h). Hint: Recall that $(\sqrt{2})^3 = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} = 2\sqrt{2}$.

(k) Extra credit: Explain (geometrically) why your answer to part (c) or (e), plus twice your answer to part (h) or (j), plus 2π, should equal $\frac{4}{3}\pi(\sqrt{2})^3 = \frac{8\sqrt{2}}{3}\pi$. Check that your answers satisfy this.