

Midterm 1

Name _____

No calculators, notes, or cheating. Each problem is worth 10 points.

1. §5.2 #33:

(a) Sketch the line $y = \frac{1}{2}x - 1$.

(b) Find $\int_0^3 (\frac{1}{2}x - 1) dx$ geometrically, using triangles.

(c) Find $\int_0^3 (\frac{1}{2}x - 1) dx$ in the usual algebraic way, using anti-derivatives.

2. This problem is about the fundamental theorem of calculus.

For a positive number x , let

$$F(x) = \int_0^x (t + 2) dt.$$

- (a) Sketch the region whose area is given by $F(x)$.

Hint: There is a vertical line on the right whose position is x .

- (b) Find $F(x)$ geometrically, using a triangle and a rectangle.

- (c) Find $F'(x)$.

3. Find

$$\int_0^{\pi/2} 4 \sin x \, dx.$$

Bonus (2 points): Sketch the region whose area this integral computes.

4. Find

$$\int_0^{\pi/2} \sin 4x \, dx.$$

Bonus (2 points): Sketch the region whose area this integral computes.

5. Based on §5.5 #43: Find

$$\int_0^1 x \sqrt[3]{1+7x^2} dx.$$

Hint: Substitute $u = 1 + 7x^2$.

To clean up at the end, recall that $8^{1/3} = \sqrt[3]{8} = 2$.

6. Find

$$\int_0^{\pi} x \cos x \, dx.$$

Hint: Integrate by parts, letting $u = x$ and $v' = \cos x$.

7. Find

$$\int_1^2 x \ln x \, dx.$$

Hint: Integrate by parts, letting $u = \ln x$ and $v' = x$.

To clean up at the end, recall that $\ln 1 = 0$.

8. Find

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx.$$

Hint: Use the identity $\cos^2 x = 1 - \sin^2 x$. Substitute $u = \sin x$.