

Midterm 2

Name _____

Each *part* of a problem is worth 5 points.

1. (a) Split up

$$\frac{3}{2x^2 + x - 1} = \frac{3}{(2x - 1)(x + 1)}$$

into partial fractions, as in §5.7.

- (b) Evaluate

$$\int_1^2 \frac{3}{2x^2 + x - 1} dx.$$

2. (a) Sketch the parabolas $y = \frac{1}{4}x^2$ and $y = 5 - x^2$.

(b) Find the two points where they meet

(c) Sketch a thin vertical slice of the region enclosed by the two parabolas. Find the height of this slice (as a function of x).

(d) Find the area of the region by evaluating an integral that involves your answer to part (c).

- (e) Sketch a thin horizontal slice of the region. Find the width of this slice (as a function of y).

Hint: You will have one answer for the lower part of the region, and a different answer for the upper part.

- (f) Find the area of the region by evaluating an integral that involves your answer to part (e). (Really it's two integrals, because your answer to part (e) had two cases.)

(g) What sanity checks can you do to see if your answers to parts (d) and (f) are plausible?

3. (a) Sketch the solid obtained by revolving the region from problem 2 around the y -axis.
Optional: Compare it to a vegetable, toy, or other familiar object.

(b) If you take the vertical slice from 2(c) and revolve it around the y -axis, it makes a thin cylindrical shell. Find the area of the side (as a function of x).

(c) Set up, *but do not evaluate*, an integral to find the volume of the solid using cylindrical shells.

- (d) If you take the horizontal slice from 2(e) and revolve it around the y -axis, it makes a thin disc. Find the area of the disc (as a function of y , again treating the lower and upper parts separately).

Hint: Be careful about radius versus diameter.

- (e) Set up, *but do not evaluate*, an integral to find the volume of the solid using discs. (Again it's really a sum of two integrals.)