Math 253

Homework 9

Due Friday, March 15, 2024

1. The function , called the sine integral, has applications to signal processing.  
   1. Try to evaluate this integral in the usual way, by making a u-substitution, integrating by parts, or whatever you can think of. Explain what you tried and where you got stuck. If you think you’ve succeeded, double-check by taking the derivative of your answer.
   2. We have seen that the Taylor series for is

Divide by t and integrate from 0 to x to get the Taylor series for .

* 1. Use this to get an approximate value for . The true value is

1. Approximating . In lecture we took the geometric series ,  
     
   substitued to get ,  
     
   and integrated to get the Taylor series for arctangent:  
     
   .  
     
   Then we noticed that , so we plugged in x=1 and multiplied by 4 to get an approximation to . But the series converged very slowly: after 100 terms we only had to one decimal place.  
     
   We also noticed that and wondered if this would yield a better approximation to .  
   Use a calculator to to plug into the Taylor series for arctangent above, carry it out to 4 terms and then 5 terms, and multiply by 6. How good an approximation do you get to ?  
     
   (Continued on the next page.)
2. This problem asks you to solve the differential equation using power series.  
   1. Suppose that   
      Find and .
   2. By equating the constant terms, then the coefficients of t, then the coefficients of and so on, solve for , , and so on in terms of .
   3. Take what you found in part (b) and write out the power series for that solves the differential equation.
   4. By know you know the Taylor series for :  
        
      .  
        
      Substitute and notice that your answer to part (c) is . Check that this satisfies .
3. “Bessel functions” appear often in applied mathematics and physics, especially in analyzing the vibration of bells, cymbals, and drums. The Bessel function of order 0 is a solution to the differential equation .  
   1. Suppose that   
      Find , , and .
   2. Add them up to get , grouping together the constant terms, the terms involving t, the terms involving , and so on; stop at .
   3. Setting your answer to part (b) equal to zero determines the coefficients , , and so on in terms of . Set the constant term equal to zero and solve for . Set the the coefficient of t equal to zero and solve for in terms of . Keep going up to , which determines .
   4. Suppose that satisfies the differential equation and the initial condition (which determines ). Write out the sixth Taylor polynomial of . Give an approximation to .