Midterm 2

Math 253

March 1, 2024 Name: Solutions

Each problem is worth 10 points, for a total of 60 points.

You may use a hand-written sheet of notes.

Show your work where appropriate.

No calculators or cheating.

1. Does converge or diverge, and why?  
     
   We can use the basic comparison test: we see that , so , and we know that diverges by the integral test (because ), so diverges as well.  
     
   Alternatively we could use the limit comparison test.

1. Does converge or diverge, and why?

This is similar to section 5.4 problem 207, which was the quiz on 2/14, but this one is a little simpler because the log isn’t squared.  
We use the limit comparison test to compare it to . We have  
  
,  
  
which is of the form , so by L’Hôpital’s rule it equals  
  
.  
  
Thus is eventually smaller than . Now converges by the integral test (because 3/2 > 1), so converges as well.  
  
Alternatively you could compare to , or for any p between 1 and 2.

1. Does converge absolutely, conditionally, or not at all, and why?  
     
   Taking absolute values, we get , which converges by the integral test (because ). So it converges absolutely.
2. Does converge or diverge, and why?  
     
   We use the ratio test:  
     
   ,  
     
   where toward the end we used the fact that . Because , the series converges.
3. Find the third Taylor polynomial of the function , that is,  
   the polynomial of degree 3 whose value and first three derivatives at zero agree with those of f.  
     
   We can take three derivatives using the chain rule:  
     
   ,  
   ,  
   .  
     
   Plugging in to f and its derivatives, we get  
     
   ,  
   ,  
   ,  
   .  
     
   Thus the third Taylor polynomial is ,  
     
   or if you want to clean it up, .
4. For which values of x does the series converge?  
     
   This was problem 12 on homework 6.  
     
   First we take absolute values and apply the ratio test. We have  
     
   .  
     
   As we see that , either using L’Hôpital’s rule or by multiplying top and bottom by , so the whole limit is . Thus if then the series converges absolutely, if then the terms don’t go to zero and the series diverges, and if then the ratio test is inconclusive.  
     
   Looking closer at the last case, if then we’re talking about the harmonic series which we know diverges, and if then we’re talking about the alternating series which converges because the absolute values of the terms decrease to zero.