Binomial Theorem

$$(x+y)^2 = (x+y)(x+y)$$

$$= x^2 + xy + yx + y^2$$

$$= x^2 + 2xy + y^2$$

$$(x+y)^3 = (x^2 + 2xy + y^2)(x+y)$$

$$= x^3 + 2x^2y + xy^2$$

$$+ x^2y + 2xy^2 + y^3$$

$$= x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x+y)^4 = (x^3 + 3x^2y + 3xy^2 + y^3)(x+y)$$

$$= x^4 + 3x^3y + 3x^2y^2 + xy^3$$

$$+ x^3y + 3x^2y^2 + 3xy^3 + y^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Pascal's Triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1 } same process

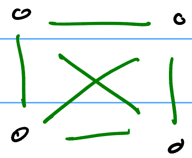
On worksheet: $(x+y)^5$.

Binomial Coefficients:

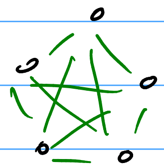
Define $\binom{n}{k}$ = for a set with n elements, how many subsets with k elements?

"n choose k"

Example: $\binom{4}{2} = 6$



$\binom{5}{2} = 10$



Formula: $\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 1}$ ← k terms here

$= \frac{n!}{k!(n-k)!}$

Example: $\binom{5}{2} = \frac{5\cdot 4}{2\cdot 1} = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{2\cdot 1\cdot \cancel{3\cdot 2\cdot 1}}$

Also: $\binom{n}{k}$ is the k^{th} entry in the n^{th} row of Pascal's △ counting from 0.

Binomial thm: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$.