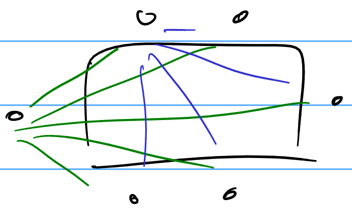
Worksheet from last time:

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
\longrightarrow 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad \boxed{56} \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\
1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1
\end{array}
$$

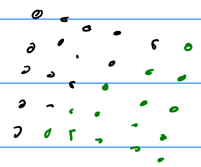
$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$\binom{8}{3} = \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3} \cdot 2 \cdot 1} = 56$$

6 people clinking glasses $\qquad \binom{6}{2} = \frac{6 \cdot 5}{2 \cdot 1} = 15$



$$5+4+3+2+1 = 15$$



$$= \frac{1}{2}(5 \times 6)$$

$$1+2+ \cdots + 100 = \binom{101}{2} = \frac{101 \cdot 100}{2 \cdot 1} = 101 \cdot 50 = 5050$$

Challenge: $(1+x)^{1/2}$?

$$1^{1/2} x^0 + \binom{1/2}{1} 1^{-1/2} x^1 + \binom{1/2}{2} 1^{-3/2} x^2 + \binom{1/2}{3} 1^{-5/2} x^3 + \dots$$

$$\binom{1/2}{2} = \frac{1/2\,(1/2 - 1)}{2 \cdot 1} = -1/8$$

$$\binom{1/2}{3} \quad \frac{1/2\,(1/2 - 1)(1/2 - 2)}{3 \cdot 2 \cdot 1} = +1/16 \text{ ?}$$

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$$1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \dots$$

Taylor series for $(1+x)^{1/2}$!

Greatest Common Divisors

Notation: $d \mid a$ ("d divides a")

means $a = md$ for some integer m

example: $12 \mid 72$ $\qquad$ TeX: \mid

$12 \nmid 64$ $\qquad$ \nmid

Division: for every $a \in \mathbb{Z}$
and every $b \in \{1, 2, 3, \ldots\}$

there exist $q \in \mathbb{Z}$ and $r \in \mathbb{Z}$.
with $0 \le r < b$

such that $a = \underbrace{q}_{\text{quotient}}b + \underbrace{r}_{\text{remainder}}$

Example:

$$\begin{array}{rl} & 5 \\ 12 & \overline{)\,64} \\ & 60 \\ \cline{2-2} & 4 \end{array}$$

$64 = 5 \cdot 12 + 4.$

if $a = 64$ and $b = 12$
then $q = 5$ and $r = 4$
in the theorem above

Greatest common divisor (gcd) of 18 and 30?

factor: $18 = 2 \cdot 3 \cdot 3$
$30 = 2 \cdot 3 \cdot 5$

So $\gcd(18, 30) = 2 \cdot 3 = 6$

UK: highest common factor (hcf)

Consider the set of $\mathbb{Z}$-linear combinations of 18 and 30

$$S = \{18m + 30n \mid m, n \in \mathbb{Z}\}$$

	m = -3	-2	-1	0	1	2	3
n = 3	36	54	72	90	108	126	144
2	6	24	42	60	58	96	114
1	-24	-6	12	30	48	66	84
0	-54	-36	-18	0	18	36	54
-1	-84	-66	-48	-30	-12	6	24
-2	-114	-96	-78	-60	-42	-24	-6
-3	-144	-126	-118	-90	-72	-54	-36

$S = \{0, \pm 6, \pm 12, \pm 18, \pm 24, \pm 30, \ldots\}$

is it just all multiples of 6?

every element of S is a mult. of 6:

$$18m + 30n = 6(3m + 5n)$$

$6 \in S$. $\quad 6 = 18 \cdot 2 - 30 \cdot 1$

every multiple of 6 is in S:

$$6k = 18 \cdot (2k) + 30(-k)$$