Comments on HW:
Write in sentences + paragraphs (with some math in them).

If you want to prove that

$$\frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2}$$

You may be tempted to write

We want to show that $\frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2}$,

which is equivalent to $\frac{n^2+n}{2} + n+1 = \frac{n^2+3n+2}{2}$,

which is equivalent to $\frac{n^2}{2} + \frac{n}{2} + n+1 = \frac{n^2}{2} + \frac{3}{2}n + 2$, ✓

which is true.

Better:

$$\begin{aligned}\frac{n(n+1)}{2} + (n+1) &= \frac{n^2+n}{2} + n+1 \\ &= \frac{n^2}{2} + \frac{n}{2} + n+1 \\ &= \frac{n^2}{2} + \frac{3}{2}n + 2 \\ &= \frac{n^2+3n+2}{2} \\ &= \frac{(n+1)(n+2)}{2}.\end{aligned}$$

Divisibility Criteria

☆ a number is even iff the last digit is even

if $n = 10q + r$ with $0 \le r \le 9$

(example: if $n = 34944$
then $q = 3494$ and $r = 4$)

know that $10 \equiv 0 \pmod 2$

so $n \equiv 0q + r \pmod 2$
$\equiv r$

So n is even
$\iff n \equiv 0 \pmod 2$
$\iff r \equiv 0 \pmod 2$
$\iff r$ is even

☆ similarly, n is a mult. of 5
iff last digit is 0 or 5

because $10 \equiv 0 \pmod 5$

so if $n = 10q + r$

then $n \equiv r \pmod 5$
so $5 \mid n$ iff $5 \mid r$
if $0 \le r \le 9$ then $5 \mid r$ iff $r = 0$ or 5.

☆ n is a mult. of 3
iff $\sum$ digits is a mult. of 3.

why? ... because $10 \equiv 1 \pmod 3$

$$\begin{aligned} 34944 &= 3\cdot 10^4 + 4\cdot 10^3 + 9\cdot 10^2 + 4\cdot 10 + 4 \\ &\equiv 3\cdot 1^4 + 4\cdot 1^3 + 9\cdot 1^2 + 4\cdot 1 + 4 \pmod 3 \\ &= 24 \end{aligned}$$

so $3 \mid 34944 \iff 34944 \equiv 0 \pmod 3$
$\iff 24 \equiv 0 \pmod 3$
which it is.

More generally, if the decimal digits of a number n are
$a_k\, a_{k-1} \cdots a_2\, a_1\, a_0$

i.e. if $n = a_k \cdot 10^k + a_{k-1}\, 10^{k-1} + \cdots + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0$

where $0 \le a_i \le 9$ for all i

then $n \equiv a_k + a_{k-1} + \cdots + a_2 + a_1 + a_0 \pmod 3$

because $10 \equiv 1 \pmod 3$

☆ similar story for 9,
because $10 \equiv 1 \pmod 9$

☆ If digits of n are $a_k \cdots a_2 a_1 a_0$

then $11 \mid n$ iff $11 \mid a_0 - a_1 + a_2 - a_3 + \cdots$

why? Because $10 \equiv -1 \pmod{11}$

So $a_k \cdot 10^k + \cdots + a_3 \cdot 10^3 + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0$

$\equiv a_k (-1)^k + \cdots + a_3 (-1)^3 + a_2 (-1)^2 + a_1 \cdot (-1) + a_0 \pmod{11}$

$= \pm a_k \mp \cdots - a_3 + a_2 - a_1 + a_0$

$+$ if k even, $-$ if k odd

☆ story for 7 ...

write $n = 10q + r$ (e.g. $34644 = 3464 \cdot 10 + 4$)

then $n \equiv n - 21r \pmod 7$

$= 10q + r - 21r$

$= 10q - 20r$

$= 10(q - 2r)$

$7 \mid n$ iff $7 \mid 10(q-2r)$ iff $7 \mid q - 2r$

by Euclid's Lemma (Prop 2.5)