Solving quadratic polynomials:

$$x^2 + 32x + 59 = 0$$

complete the $\square$: sub. $x = z - 16$

$$(z-16)^2 + 32(z-16) + 59 = 0$$

$$z^2 - \cancel{32z} + 256 + \cancel{32z} - 512 + 59 = 0$$

$$z^2 - 197 = 0$$

$$z^2 = 197$$

$$z = \pm\sqrt{197}$$

$$x = z - 16 = -16 \pm \sqrt{197}$$

In general:

to solve $ax^2 + bx + c = 0$

divide by a: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

sub. $x = z - \frac{b}{2a}$ $\quad z^2 - \cancel{\frac{b}{a}z} + \frac{b^2}{4a^2} + \cancel{\frac{b}{a}z} - \frac{b^2}{2a^2} + \frac{c}{a} = 0$

$$z^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$z^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

$$z = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x = z - \frac{b}{2a} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Solving cubic polynomials.

example 2: $x^3 - 6x^2 + 6x - 2 = 0$

eliminate $-6x^2$ by subbing $x = z + 2$

$$z^3 + \cancel{6z^2} + 12z + 27 - 6(\cancel{z^2} + 4z + 4) + 6(z+2) - 2$$

$$z^3 - 6z - 6 = 0$$

$$z^3 + pz + q = 0$$

Viète's trick: sub $z = v + \frac{2}{v}$

$$z = v - \frac{p}{3v}$$

$$z^3 = v^3 + 3v^2\left(\frac{2}{v}\right) + 3v\left(\frac{2}{v}\right)^2 + \left(\frac{2}{v}\right)^3$$

$$= v^3 + 6v + \frac{12}{v} + \frac{8}{v^3}$$

$$z^3 - 6z - 6 = v^3 + \cancel{6v} + \cancel{\frac{12}{v}} + \frac{8}{v^3}$$

$$- \cancel{6v} - \cancel{\frac{12}{v}} \qquad - 6$$

$$= v^3 - 6 + \frac{8}{v^3} \quad (= 0)$$

mult. thru by v^3 to get

$$(v^3)^2 - 6v^3 + 8 = 0$$

$$u = v^3$$

$$u^2 + 6u + 8$$

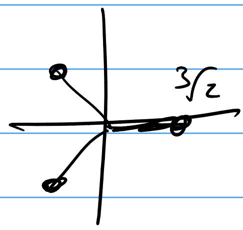
$$\omega = e^{2\pi i/3} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$\omega^3 = 1 \qquad \frac{1}{\omega} = \omega^2$$

$$(v^3 - 2)(v^3 - 4) = 0$$

$$\underbrace{v^3 = 2}_{} \quad \text{or} \quad \underbrace{4}_{} \to \underline{3 \text{ more}}$$

$$v = \sqrt[3]{2} \quad \text{or} \quad \sqrt[3]{2}\,\omega \quad \text{or} \quad \sqrt[3]{2}\,\omega^2$$



Sub. back:

$$z = v + \frac{2}{v} = \sqrt[3]{2} + \frac{2}{\sqrt[3]{2}} = \sqrt[3]{2} + \sqrt[3]{4}$$

$$\text{or } z = \sqrt[3]{2}\,\omega + \frac{2}{\sqrt[3]{2}\,\omega} = \sqrt[3]{2}\,\omega + \sqrt[3]{4}\,\omega^2$$

$$\text{or } z = \sqrt[3]{2}\,\omega^2 + \frac{2}{\sqrt[3]{2}\,\omega^2} = \sqrt[3]{2}\,\omega^2 + \sqrt[3]{4}\,\omega$$

$$x = z + 2 = 2 + \sqrt[3]{2} + \sqrt[3]{4} \quad \text{or}$$

$$2 + \sqrt[3]{2}\,\omega + \sqrt[3]{4}\,\omega^2$$

$$2 + \sqrt[3]{2}\,\omega^2 + \sqrt[3]{4}\,\omega$$

example ①
$$z^3 - 3z + 1 = 0$$

$$\text{sub } z = v + \frac{1}{v}$$

$$v^3 + \cancel{3v^2\frac{1}{v}} + \cancel{3v\frac{1}{v^2}} + \frac{1}{v^3} - \cancel{3v} - \cancel{3\frac{1}{v}} + 1$$

$$v^3 + 1 + \frac{1}{v^3} = 0$$

$$(v^3)^2 + v^3 + 1 = 0 \qquad v^3 = \frac{-1 \pm \sqrt{-3}}{2}$$

out of time