

# Worksheet 13

Math 391, Abstract Algebra

Friday, October 30, 2020

Consider the numbers

$$0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, 1, 2, 3.$$

1. Let  $S = \{0, \frac{1}{2}, 1\} \subset \mathbb{Q}$ .

Of the numbers above, which ones are upper bounds for  $S$ ? Does  $S$  have a least upper bound in  $\mathbb{Q}$ ? If so, what is it? If not, what is its least upper bound in  $\mathbb{R}$ ?

2. Let  $S = \{x \in \mathbb{Q} : x^2 < \frac{1}{4}\}$ .

Draw a picture of  $S$ . Show your picture to your colleagues, either by using the Zoom whiteboard or by holding up a pad of paper to your camera. Be prepared to show it to me or Elisa when we visit!

Then answer the same questions as in #1.

3. Let  $S = \{x \in \mathbb{Q} : x^2 \leq \frac{1}{4}\}$ .

Draw a picture of  $S$  and answer the same questions.

4. Let  $S = \{x \in \mathbb{Q} : x^2 \leq \frac{1}{2}\}$ .

Draw a picture of  $S$  and answer the same questions.

5. Let  $S = \{x \in \mathbb{Q} : x^2 \geq \frac{1}{2}\}$ .

Draw a picture of  $S$  and answer the same questions.

6. Challenge: Let  $b$  be the least upper bound that you found in #2. Prove carefully that it's really the least upper bound for  $S$ . There are two things to prove:

- (a) It's an upper bound: for all  $s \in S$  we have  $s \leq b$ .
- (b) Any smaller number is *not* an upper bound: for every  $a < b$ , there is an  $s \in S$  such that  $a < s$ .