Worksheet 14
Math 391, Abstract Algebra
Monday, November 2, 2020

Just do as many of these as you have time for.

1. Let $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$.
   
   (a) Draw $z$ as a point in the complex plane.
   (b) Find a radius $r$ and angle $\theta$ such that $z = r(\cos \theta + i \sin \theta)$.
   (c) Find a square root of $z$: that is, a complex number $w$ such that $w^2 = z$. Your final expression for $w$ should not have any sines or cosines in it.
   (d) Take the number $w$ that you found in part (c) and compute $w^2$ by hand to make sure that you get back $z$.

   Notice that $z$ actually has two square roots: $w$ and $-w$.

2. Same for the square root of $i$.

3. Now find a cube root of $i$: that is, find a $w$ such that $w^3 = i$.

   In fact you can find three such $w$’s. To see this, notice that $90^\circ$ and $450^\circ$ and $-270^\circ$ all represent the same angle, but when you divide them by 3 you get $30^\circ$ and $150^\circ$ and $-90^\circ$ which represent different angles.

4. Challenge: Find the square roots of $z = 1 + 2i$. To eliminate trig functions from your expression for $w$, you can use the half-angle formulas

   $\cos^2(\theta/2) = \frac{1 + \cos \theta}{2}$  \hspace{1cm}  $\sin^2(\theta/2) = \frac{1 - \cos \theta}{2}$. 