Worksheet 14

Math 391, Abstract Algebra

Monday, November 2, 2020

Just do as many of these as you have time for.

1. Let $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

   (a) Draw z as a point in the complex plane.

   (b) Find a radius r and angle θ such that $z = r(\cos\theta + i\sin\theta)$.

   (c) Find a square root of z: that is, a complex number w such that $w^2 = z$. Your final expression for w should not have any sines or cosines in it.

   (d) Take the number w that you found in part (c) and compute w^2 by hand to make sure that you get back z.

   Notice that z actually has two square roots: w and $-w$.

2. Same for the square root of i.

3. Now find a cube root of i: that is, find a w such that $w^3 = i$.

   In fact you can find three such w's. To see this, notice that 90° and 450° and -270° all represent the same angle, but when you divide them by 3 you get 30° and 150° and -90° which represent different angles.

4. Challenge: Find the square roots of $z = 1 + 2i$. To eliminate trig functions from your expression for w, you can use the half-angle formulas

$$\cos^2(\theta/2) = \frac{1+\cos\theta}{2} \qquad \sin^2(\theta/2) = \frac{1-\cos\theta}{2}.$$