

Worksheet 3

Math 391, Abstract Algebra

Monday, October 5, 2020

This is based on Shifrin §1.2 #1.

0. Introduce yourself to your colleagues. What are their favorite primes?
1. In lecture we discussed the Euclidean algorithm for finding $d = \gcd(a, b)$:
 - (i) Assume that $a > b$; otherwise switch them. Divide a by b to get a quotient q_1 and a remainder r_1 .
 - (ii) Divide b by r_1 to get a quotient q_2 and a remainder r_2 .
 - (iii) Divide r_1 by r_2 to get a quotient q_3 and a remainder r_3 .
 - (iv) Keep going until you get a remainder of zero. Then d is the last non-zero remainder that you got.

Use the Euclidean algorithm to find the greatest common divisor of each of the following pairs of numbers. Work together, and take turns doing the arithmetic. In each case, try to understand why the algorithm gave the right answer, rather than just believing it.

- (a) 14, 35
 - (b) 56, 77
 - (c) 618, 336
 - (d) 2873, 6643
 - (e) 512, 360
 - (f) 4432, 1080
2. Challenge: Go back and find integers m and n such that $d = ma + nb$ in each case. Try to use q_i s and r_i s to find m and n systematically, rather than just guessing.