

# Worksheet 4

Math 391, Abstract Algebra

Wednesday, October 7, 2020

This is based on Shifrin §1.3 #9, and is a good warm-up for §1.3 #15 on next Monday's homework.

0. Introduce yourself to your colleagues. Do they play any musical instruments?
1. In lecture we showed that for any integer  $n$ , we have  $n^2 \equiv 0, 1, 4, 5, 6,$  or  $9 \pmod{10}$ . Show that  $n^4 \equiv 0, 1, 5,$  or  $6 \pmod{10}$ .
2. Show that if  $N$  is a sum of two fourth powers – that is, if we can write  $N = m^4 + n^4$  for some integers  $m$  and  $n$  – then  $N \equiv 0, 1, 2, 5, 6,$  or  $7 \pmod{10}$ .
3. What about a sum of three fourth powers?
4. Can  $232^2 = 53824$  be written as a sum of three fourth powers?
5. Challenge: What can you say about last digits of cubes? Fifth powers?