Last worksheet: $G$: notation of the cube.

Looked at elements of order 1, 2, 3, 4.

How many of each?

Turn it into a homework problem

Been talking about $S_n$

In $S_5$: 2 1 2 3 4 5

$\tau = (1 2 3 4 5)$

$\sigma = (2 1) \sigma^2 = 1$ so $|\tau| = 2$

$\circ = 2 3 4 5$

$\sigma = (2 1 3 4 5)$

Some people: $(2 1 3 4 5)$

Cycle notation: $(1 2)$

$\sigma = 1 2 3 4 5 \sigma^2 = 1$ $\sigma^3 = 1$

So $|\sigma| = 3$

$\circ = 2 3 4 5$

Cycle notation: $(3 4 5)$

$\sigma^2 = (3 5 4)$
More examples of groups:

If $R$ is a ring, then the additive group of $R$ is just $R$, operation $+$, forget about mult.

Group axioms:
1. $(a+b)+c = a+(b+c)$
2. Identity $e = 0$
   (see $a+0 = 0+a = a$)
3. Inverses:
   $a + (-a) = (-a)+a = 0$

In fact it's an Abelian group: $a+b = b+a$.

Favorites: $\mathbb{F}_2$, $\mathbb{F}_3$, $\mathbb{F}_2 \times \mathbb{F}_2$, "k(x) 4-group"

as groups under $+$,

$\mathbb{F}_2 \times \mathbb{F}_2$ and $\mathbb{F}_2[x]/x$ and $\mathbb{F}_2[x]/x^2 + x + 1$

are isomorphic (though not as rings.)

$\mathbb{F}_2 \times \mathbb{F}_2$ and $\mathbb{F}_2[x]/x$

$(0,0) \leftrightarrow 0$ this bijection respects $+$
$(1,0) \leftrightarrow x$ but not $x$
$(0,1) \leftrightarrow x+1$
$(1,1) \leftrightarrow 1$
the Dihedral group $D_n = \text{symmetries of a regular n-gon that contain a subgroup isomorphic to the additive group of \( \mathbb{Z}_n \)}$

if $\theta = \text{rotate 72}^\circ$

$\mathbb{Z}_5 \rightarrow D_5$

\[
\begin{align*}
0 & \rightarrow 1 \\
1 & \rightarrow \theta \\
2 & \rightarrow \theta^2 \\
3 & \rightarrow \theta^3 \\
4 & \rightarrow \theta^4 \\
5 \equiv 0 & \rightarrow \theta^5 = 1
\end{align*}
\]

not the whole of $D_5$ - missing the reflections.

if $R$ is a ring, let $R^x = \{\text{units in } R\}$

these form a group under multiplication.

example: $\mathbb{Z}^x = \{1, -1\}$ under multiplication

\[
\mathbb{Z}^x \times \mathbb{Z}^x = \{1, 1, -1, -1\} \cong (\mathbb{Z}_2, +)
\]

Worksheet: $\mathbb{Z}^x_{10}, \mathbb{Z}^x_{12},...$