Last worksheet: G = rotations of the cube.
looked elements of order 1, 2, 3, 4.
how many of each?
↳ turn it into a homework problem

Been talking about S_n

In S_5: τ

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \multicolumn{2}{c}{\times} & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 \end{array}$$

$\tau \neq 1$ but $\tau^2 = 1$ so $|\tau| = 2$

1 2 3 4 5

Shifrin: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix}$

Some people: $(2\ 1\ 3\ 4\ 5)$

Cycle notation: $(1\ 2)$

$$\sigma = \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \multicolumn{3}{c}{\times} \\ 1 & 2 & 3 & 4 & 5 \end{array}$$

$\sigma \neq 1$
$\sigma^2 \neq 1$
$\sigma^3 = 1$
so $|\sigma| = 3$

1 2 3 4 5

cycle notation: $(3\ 4\ 5)$

$\sigma^2 = (3\ 5\ 4)$

More examples of groups:

if R is a ring, then the <u>additive group</u> of R

is just R, operation $= +$, forget about mult.

Group axioms? ① $(a+b)+c = a+(b+c)$

② identity $e = 0$

bec. $a+0 = 0+a = a \quad \forall a \in R$

③ inverses:

$a+(-a) = (-a)+a = 0$

In fact it's an Abelian group. $a+b = b+a$.

Favorites: $\mathbb{Z}$, $\mathbb{Z}_m$ $\quad \mathbb{Z}_2 \times \mathbb{Z}_2$ "Klein 4-group"

as groups under $+$,

$\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_2[x]/x^2$ and $\mathbb{Z}_2[x]/x^2+x+1$

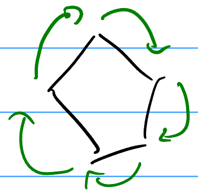
are isomorphic (though not as rings!)

$\mathbb{Z}_2 \times \mathbb{Z}_2$		$\mathbb{Z}_2[x]/x^2$
$(0,0)$	$\longleftrightarrow$	0
$(1,0)$	$\longleftrightarrow$	x
$(0,1)$	$\longleftrightarrow$	$x+1$
$(1,1)$	$\longleftrightarrow$	1

this bijection respects $+$ but not $\times$.

the Dihedral group D_n = symmetries of a regular n-gon

contains a subgroup isomorphic to (the additive group of) $\mathbb{Z}_n$



if r = rotate $72°$

$$\begin{aligned}
\mathbb{Z}_5 &\longrightarrow D_5 \\
0 &\longmapsto 1 \\
1 &\longmapsto r \\
2 &\longmapsto r^2 \\
3 &\longmapsto r^3 \\
4 &\longmapsto r^4 \\
5=0 &\longmapsto r^5 = 1
\end{aligned}$$

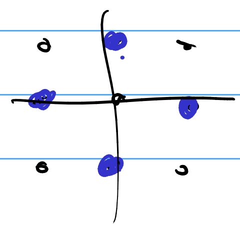
not the whole of D_5 – missing the reflections.

if R is a ring, let
$R^\times = \{\text{units in } R\}$

then form a group under multiplication.

example: $\mathbb{Z}^\times = \{1, -1\}$ under multiplication

$\underline{\underline{\mathbb{Z}[i]^\times}} = \{1, i, -1, -i\} \cong (\underline{\underline{\mathbb{Z}_4}}, +)$



$$\begin{aligned}
& \varphi \\
1 &\longleftarrow 0 \\
i &\longleftarrow 1 \\
-1 &\longleftarrow 2 \\
-i &\longleftarrow 3
\end{aligned}$$

$$\begin{aligned}
\varphi(m+n) &= i^{(m+n)} \\
&= i^m i^n \\
&= \varphi(m)\varphi(n)
\end{aligned}$$

Worksheet: $\mathbb{Z}_{10}^\times$, $\mathbb{Z}_{12}^\times$, ...