On one hand, let \( G \) act on a set \( X \)
let \( x \in X \), let \( H = G_x = \text{the stabilizer of } x \).

the map \[ G \rightarrow X \]
\[ g \rightarrow gx \]
gives a bijection between
left cosets of \( H \)
and elements of the orbit \( O_x \):

\[ \forall y \in O_x, \quad \{ g \in G \mid gx = y \} \]
is a coset of \( H \)

Lagrange: \[ |H| \cdot \# \text{ cosets} = |G| \]
so \[ |\text{stabilizer} \cdot \text{orbit}| = |G| \]

On the other hand, given a subgroup \( H < G \),
let \( G \) act on \( \{ \text{subsets of } G \} \) by left mult.
orbite of \( H = \{ \text{all left cosets } gH \} \)
and stabilizer of \( H = H \) itself.

if we know that \[ |\text{orbit}| \cdot |\text{stabilizer}| = |G| \]
then \[ \# \text{ cosets} \cdot |H| = |G| \] (Lagrange)

An action of \( G \) on any set \( H \)
breaks up into orbits,
and action on each orbit is "the same" as
action of \( G \) on \( \{ \text{cosets of a stabilizer} \} \).
Kept coming up: \((gh)^{-1} = h^{-1} g^{-1}\)

because \((gh)(h^{-1} g^{-1}) = g(h h^{-1}) g^{-1} = g g^{-1} = 1\)

but \(gh.g^{-1}\) is probably not \(= 1\).

(Unless \(gh = hg\)).

Look at \(D_6\) acting on itself by conjugation:

\[
D_6 = \{ 1, \ r, \ r^2, \ r^3, \ r^4, \ r^5, \ s, \ sr, \ sr^2, \ sr^3, \ sr^4, \ sr^5 \}
\]

\[
sr = sr^{-1}
\]

\(g\) sends \(x\) to \(g x g^{-1}\) ....

Orbits: \[
\begin{align*}
\{ 1 \} & \check \quad \{ r, r^{-1} \} & \check \\
\{ r^2, r^{-2} \} & \check \\
\{ r^3 \} & \check \\
\{ s, sr^2, sr^4 \} & \check \\
\{ sr, sr^3, sr^5 \} & \check
\end{align*}
\]

Calc: ① if \(g = r^n\) and \(x = r^m\)

then \(g x g^{-1} = r^n r^m r^{-n} = r^m\)

② if \(g = sr^n\) and \(x = r^m\)

then \(g x g^{-1} = sr^n r^m r^{-n} s\)

\[
= sr^n s
\]

\[
= sr^n s r^{-n} = r^{-n}
\]
\(3\) if \(g = r^m\) and \(x = sr^n\) then \(gxg^{-1} = r^m \cdot sr^n \cdot r^{-m} = sr^{-m} \cdot sr^n \cdot r^{-m} = sr^{n-2m}\)

\(4\) if \(g = sr^m\) and \(x = sr^n\) then \(gxg^{-1} = sr^m \cdot sr^n \cdot s^{-m}\)

\(= sr^m \cdot s^{-m} \cdot sr^n = sr^m \cdot sr^{-m} \cdot r^{-m} = sr^{n-2m}\)