Orbit-Stabilizer thm vs. Lagrange's theorem:

on one hand, let G act on a set X
let $x \in X$, let $H = G_x$ = the stabilizer of x.

the map
$$\begin{aligned} G &\longrightarrow X \\ g &\longmapsto gx \end{aligned}$$

gives a bijection between
left cosets of H
and elements of the orbit $\mathcal{O}_x$:

$\forall\, y \in \mathcal{O}_x$, $\{g \in G \mid gx = y\}$
is a coset of H

Lagrange: $|H| \cdot \#\text{cosets} = |G|$
so $|\text{stab}| \cdot |\text{orbit}| = |G|$

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on the other hand, given a subgroup $H \subset G$,
let G act on $\{\text{subsets of } G\}$ by left mult.
orbit of H = $\{\text{all left cosets } gH\}$
and stabilizer of H = H itself.

if we know that $|\text{orbit}| \cdot |\text{stab}| = |G|$
then $\#\text{cosets} \cdot |H| = |G|$ (Lagrange)

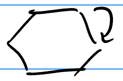
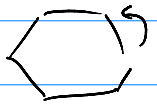
An action of G on any set H
breaks up into orbits,
and action on each orbit is "the same" as
action of G on $\{\text{cosets of a stabilizer}\}$

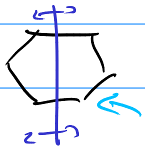
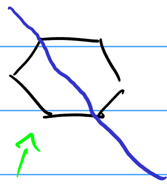
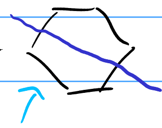
Looked at D_4, D_5 acting on themselves by conjugation.

Kept coming up: $(gh)^{-1} = h^{-1}g^{-1}$

because $(gh)(h^{-1}g^{-1}) = g(\cancel{hh^{-1}})g^{-1} = gg^{-1} = 1$
but $gh \cdot g^{-1}h^{-1}$ is probably not $= 1$.
(unless $gh = hg$).

Look at D_6 acting on itself by conjugation:

$D_6 = \{ 1,\ r =$  $60°$ $r^2,\ r^3,\ r^4,\ r^5$  $60°$

s  sr  sr^2  $sr^3\ sr^4\ sr^5 \}$

$rs = sr^{-1}$ $\qquad$ g sends x to gxg^{-1} ....

Orbits: $\{1\}$ ✓ $\quad \{r, r^{-1}\}$ ✓ $\quad \{r^2, r^{-2}\}$ ✓ $\quad \{r^3\}$ ✓

$\{\underline{s}, \underline{sr^2}, \underline{sr^4}\}$ $\qquad \{\underline{sr}, \underline{sr^3}, \underline{sr^5}\}$

Calcs: ① if $g = r^m$ and $x = r^n$
then $gxg^{-1} = r^m r^n r^{-m} = r^n$

② if $g = sr^m$ and $x = r^n$
then $gxg^{-1} = sr^m r^n r^{-m}s$
$= sr^n s$
$= s \cdot sr^{-n} = r^{-n}$

(3) if $g = r^m$ and $x = sr^n$

then $gxg^{-1} = \underbrace{r^m \cdot s}r^n \cdot r^{-m}$

$= \underbrace{sr^{-m}} \cdot r^n r^{-m} = sr^{n-2m}$

(4) if $g = sr^m$ and $x = sr^n$

then $gx\underline{g^{-1}} = sr^m \cdot sr^n \cdot \underline{r^{-m}s}$

$= sr^m \, s\underbrace{r^n \, s}r^m$

$= sr^m \cancel{s\,s}\underbrace{r^{-n}} r^m \qquad s^2 = 1$

$= s r^m r^{-n} r^m = sr^{2m-n}$