

Homework 7

Due Friday, February 19, 2021

Continued from Worksheet 12, which accompanied Lecture 16. Let G be the group of rotations of the cube. We have seen that $|G| = 24$.

1. How many elements of order 4 are there in G ? That is, how many 90° rotations? Describe them.
2. How many elements of order 3, or 120° rotations? Describe them.
3. How many elements of order 2, or 180° rotations? Describe them. Notice that there are two different kinds.
4. How many elements of order 1? Do your numbers add up to 24?
5. Label the six faces of a cube as 1, 2, 3, 4, 5, 6 in whatever way you want. Choose some 90° rotation. It gives a permutation of the six faces: maybe it moves face 1 to where face 2 used to be, face 2 to where face 4 used to be, and so on. Write down the permutation, using whatever notation you like. Of course your answer will depend on how you labeled the faces, and on which rotation you chose.
(Sanity check: Does your permutation have order 4?)
6. Label the eight vertices as a, b, c, d, e, f, g, h , again in whatever way you want. The same 90° rotation permutes these as well. Write down the permutation.
7. Same with a 120° rotation: choose one, and say how it permutes the faces and the vertices. Do your permutations have order 3?
8. Same with a 180° rotation – actually, do one of each kind. Do your permutations have order 2?

Also do §6.2 #12.