

Worksheet 13

Math 392, Abstract Algebra

Friday, February 12, 2021

1. Here is the additive group of \mathbb{Z}_{10} :

$$\{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}\}.$$

Find the order of each element.

Be careful: The group operation is $+$, not \times , so the identity element is $\bar{0}$ rather than $\bar{1}$, and the order of an element $a \in \mathbb{Z}_{10}$ is the smallest number n such that

$$\overbrace{a + a + \cdots + a}^{n \text{ times}} = \bar{0}.$$

2. The group of units \mathbb{Z}_{10}^\times has four elements. What are they? Then find the order of each one.

Be careful: Now the group operation is \times , so the identity element is $\bar{1}$, and the order of an element $a \in \mathbb{Z}_{10}^\times$ is the smallest number n such that

$$\overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ times}} = \bar{1}.$$

3. Write down a group isomorphism between \mathbb{Z}_{10}^\times and the additive group of \mathbb{Z}_4 . So you want a bijection between \mathbb{Z}_{10}^\times and \mathbb{Z}_4 , under which multiplication in \mathbb{Z}_{10}^\times corresponds to addition in \mathbb{Z}_4 .
4. Same with \mathbb{Z}_{12} :

$$\{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \bar{10}, \bar{11}\}.$$

In this case \mathbb{Z}_{12}^\times is isomorphic to the additive group of $\mathbb{Z}_2 \times \mathbb{Z}_2$, not \mathbb{Z}_4 .

5. Challenge: Investigate \mathbb{Z}_n^\times for more values of n . Each one is isomorphic to the additive group of \mathbb{Z}_m for some m , or of $\mathbb{Z}_m \times \mathbb{Z}_{m'}$ for some m and m' , until you get up to $n = 24$.