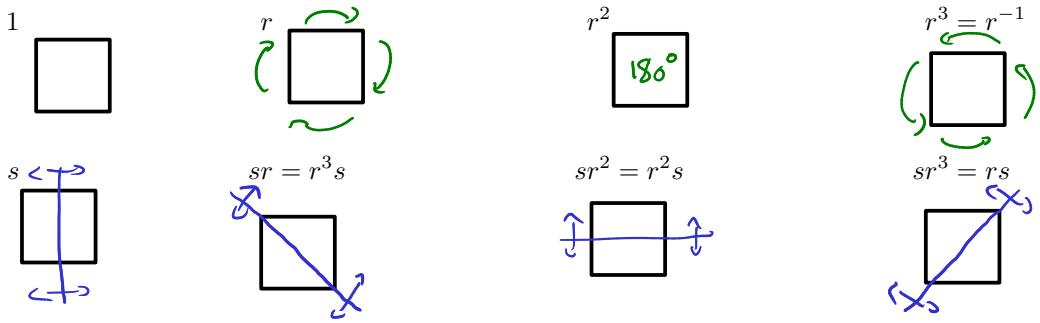
Worksheet 15

Math 392, Abstract Algebra

Wednesday, February 17, 2021

Let D_4 be the symmetry group of the square, and let r and s be the rotation and reflection that you studied last time. The group has eight elements:



Label the vertices of the square as shown:

$$\begin{array}{ccc} 1 & - & 2 \\ | & & | \\ 4 & - & 3 \end{array}$$

Let X be the set of ordered pairs of vertices, which has 16 elements:

$$\begin{array}{cccc} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4). \end{array}$$

Let D_4 act on X in the natural way, so for example, r sends $(1,2)$ to $(2,3)$, and s sends $(1,2)$ to $(2,1)$.

1. See where the various elements of D_4 send $(1,2)$. This gives you a subset of X with eight elements, called the *orbit* of $(1,2)$.
2. The orbit of $(1,3)$ only has four elements. What are they?
3. The stabilizer of $(1,3)$ is a subgroup of D_4. What are its elements? Hint: There are two.
4. Same with the orbit and stabilizer of $(1,1)$.
5. Are there any other orbits, or have you found them all?