Get out your cubes from the other day.

Let $G$ be the group of rotations of the cube. Our convention is that multiplication in $G$ works like composition of functions, so if $g$ and $h$ are two elements of $G$, then $gh$ means $g$ after $h$. We could have made the opposite choice, but we didn’t.

1. Choose two adjacent faces of your cube and call them face 1 and face 2. Let $h$ be an element of $G$ that rotates face 1 by $90^\circ$, and let $g$ be an element that moves face 1 to face 2. Of the elements
   \[ g^{-1}hg \quad ghg^{-1} \quad h^{-1}gh \quad hgh^{-1}, \]
   one of them rotates face 2 by $90^\circ$. Which one is it? Try to get a feeling for why the answer makes sense.

2. Choose two adjacent vertices of your cube and call them vertex $a$ and vertex $b$. Let $h$ be an element of $G$ that rotates vertex $a$ by $120^\circ$, and let $g$ be an element that moves vertex $a$ to vertex $b$. Of the elements
   \[ g^{-1}hg \quad ghg^{-1} \quad h^{-1}gh \quad hgh^{-1}, \]
   one of them rotates vertex $b$ by $120^\circ$. Which one is it? Does this still make sense?

3. Let $h$ be an element of $G$ that rotates face 1 by $180^\circ$, and let $k$ be an element that rotates some edge by $180^\circ$. Convince yourselves that there is no $g \in G$ such that $ghg^{-1} = k$.

4. Challenge: Let $h$ be a $90^\circ$ rotation as in problem 1. Convince yourselves that there is an element $g \in G$ such that $ghg^{-1} = h^{-1}$. How many such elements are there? Same with a $120^\circ$ rotation as in problem 2.