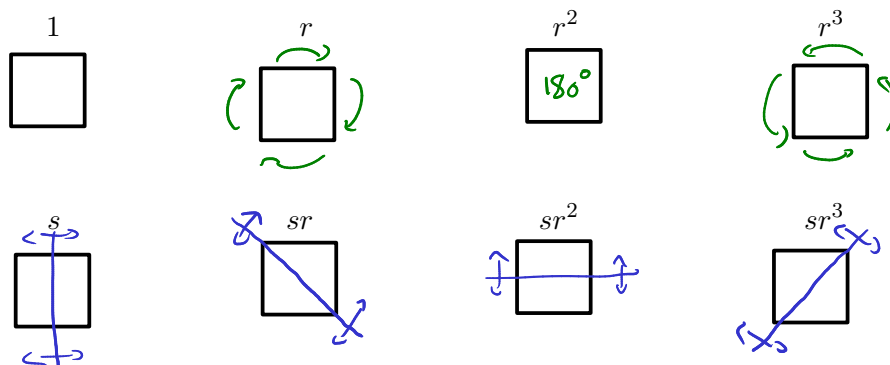


Worksheet 17

Math 392, Abstract Algebra

Wednesday, February 24, 2021

Let D_4 be the symmetry group of the square, as usual:



We have $r^4 = 1$ and $s^2 = 1$, and we've worked hard to figure out that $rs = sr^{-1}$, and more generally $r^k s = sr^{-k}$.

Let D_4 act on itself by conjugation: so $g \in D_4$ acts on $x \in D_4$ by sending it to gxg^{-1} .

1. Find the orbit of r under this action.

Hint: First take a general rotation $g = r^k$ and compute grg^{-1} . Then take a general reflection $g = sr^k$ and compute grg^{-1} . This should be similar to what happened with D_5 in lecture.

2. Find the stabilizer of r under this action.

Hint: That means set of all $g \in D_4$ such that $grg^{-1} = r$. This should follow pretty easily from your work in #1. If the orbit had order 2, then the stabilizer should have order 4.

3. Find the orbit and stabilizer of r^2 .

4. Find the orbit and stabilizer of s .

Notice that in D_5 , all the reflections were conjugate to one another, but in D_4 they're not.

5. Find the orbit and stabilizer of sr .

6. Challenge: Carry out the same analysis on D_6 , the symmetry group of a regular hexagon.