Worksheet 5

Math 392, Abstract Algebra

Wednesday, January 13, 2021

Let $R = \mathbb{Z}_2[x]$ and $I = \langle x^3 + x + 1 \rangle$.

1. Convince yourselves that R/I has eight elements:

$$\begin{array}{cccc} \bar{0} & \bar{1} & \bar{x} & \overline{x+1} \\ \overline{x^2} & \overline{x^2+1} & \overline{x^2+x} & \overline{x^2+x+1} \end{array}$$

   So there are two things to see: First, for any $f \in R$, the equivalence class $\bar{f} \in R/I$ is equal to one of those eight. And second, none of those eight are equal to each other.

2. Convince yourselves that $\overline{x^3+x+1} = \bar{0}$, and so $\bar{x}^3 = \overline{x+1}$.

   (Recall that $-1 = 1$ in $\mathbb{Z}_2$ and hence in R and R/I.)

3. Find $\bar{x}^4$, $\bar{x}^5$, $\bar{x}^6$, $\bar{x}^7$.

4. Challenge: $\bar{x}$ is a unit in R/I; find its inverse.